To The Graduate School:

The members of the Committee approve the thesis of Praveen Sodanapalli Presented on November 16, 2000.

Dennis N. Coon, Chairman

Ann Nancy W. Peck

Jerry C. Hamann

APPROVED:

William R. Lindberg, Head, Department of Mechanical Engineering

Stephen E. Williams, Dean, The Graduate School

Sodanapalli, Praveen., Simulation of Creep Crack Growth in Ceramic Composites, M.S., Department of Mechanical Engineering, December 2000

Tensile creep behavior and the elevated temperature fatigue of fiber reinforced ceramic composites were investigated using Monte Carlo simulation. The simulated model consisted of a uniaxially loaded tow of unidirectional fibers aligned parallel to the load. The simulation generated a Weibull distribution of fiber strengths, and Gaussian distributions of fiber modulus and radius. The simulation assumed a creep strain rate consisting of primary and steady state components each of which was modeled by a power law relationship. Power law exponents in the range of 0-7 for a selected SiC/SiC system at stress levels ranging from 60 MPa to 200 MPa were evaluated. A fatigue exponent of 3.03 $\pm$ 0.066 was predicted for nominal stress levels less than 150 GPa. The influence of initial crack length on the failure lifetime was also studied. A comparison of the predicted failure response and literature data suggested a stress dependent creep process could be used to model experimental data and possibly evaluate the failure mechanism of reinforced test items. Average magnitude and standard deviation of fiber characteristics were varied in the simulation while keeping the creep model constant. Low values of fiber radius seemed to increase the lifetime while higher values had little impact on the failure lifetime. Increased fracture toughness increased the lifetime of the composite for moderate values but had little effect for higher values of fracture toughness. Both characteristic strength and the Weibull modulus of the fibers were predicted to have significant effect on the creep life of the fiber tow. An increase in either the characteristic strength or Weibull modulus was predicted to result in an increase in creep life with the former having more influence than the latter. Minimizing the spread in the values of the elastic modus of the fibers may lead to an increased lifetime of the fiber tow.

SIMULATION OF CREEP CRACK GROWTH IN CERAMIC COMPOSITES

By

Praveen Sodanapalli

A thesis submitted to the Department of Mechanical Engineering and the Graduate School of the University of Wyoming in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE
in
MECHANICAL ENGINEERING

Laramie, Wyoming
December 2000

PREFACE

An investigation into the creep crack growth behavior of ceramic composites has been completed. This investigation employed a microstructural model to simulate damage growth in a fiber-reinforced ceramic induced by static loads at elevated temperatures to assess the life of a fiber tow. The material model correlated well with the experimental data as proof of concept. Next the simulation was used to predict what effects changing fiber properties had on tow life.

This investigation was performed by Praveen Sodanapalli in partial requirements for the degree of Master of Science. The investigation was undertaken at he University of Wyoming under the direction of Dr. Dennis N. Coon, Professor of Mechanical Engineering.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Section I

Theoretical Background

The resistance of brittle materials to tensile failure can be enhanced considerably by reinforcement with high strength fibers. The most dramatic improvements in properties have been achieved in composites that contain continuous, weakly bonded fibers aligned parallel to the tensile axis. The use of fibrous reinforced ceramic materials instead of monolithic ceramics in engineering structures allows higher mechanical and thermal performances, and weight reduction. This class of composites includes glasses, glass-ceramics, and ceramics reinforced by carbon and SiC fibers. Mechanisms of failure in these composites and in monolithic ceramics can differ substantially. Monolithic ceramics generally fail by the growth of a single crack on a plane normal to maximum principal stress. Fiber composites, on the other hand, can fail by a variety of mechanisms, dependent upon the applied stress state and the geometry and the microstructural characteristics of the composite. The micro-mechanisms that lead to improved fracture resistance in ceramic composites include microcrack toughening, transformation toughening, ductile phase toughening, fiber toughening and whisker toughening. Although the formation of cracks in a material is generally considered deleterious, microcracking can some times lead to improved toughness. The formation of microcracks releases strain energy from the sample, which results in toughening of the material. Many toughened ceramics contain second-phase particles that are capable of nonlinear deformation, and are primarily responsible for the elevated toughness. One of the most effective toughening mechanisms in ceramic composites is the fiber bridging mechanism [9].

Figure 1 illustrates the fiber bridging mechanism, when a propagating crack leaves fibers or second–phase particles intact. Once the matrix has cracked, the load is carried by the fibers. The fibers do not fail simultaneously because the fiber strength is subject to statistical variability. Consequently, the material exhibits quasi-ductility where damage accumulates gradually until final failure. Not only is fiber bridging the most effective toughening mechanism for ceramics, it is also effective at high temperatures [28]. Consequently applications requiring load-bearing capability at temperatures above

1000 ^{0}C would undoubtedly benefit from utilizing fiber–reinforced ceramics exhibiting fiber bridging.

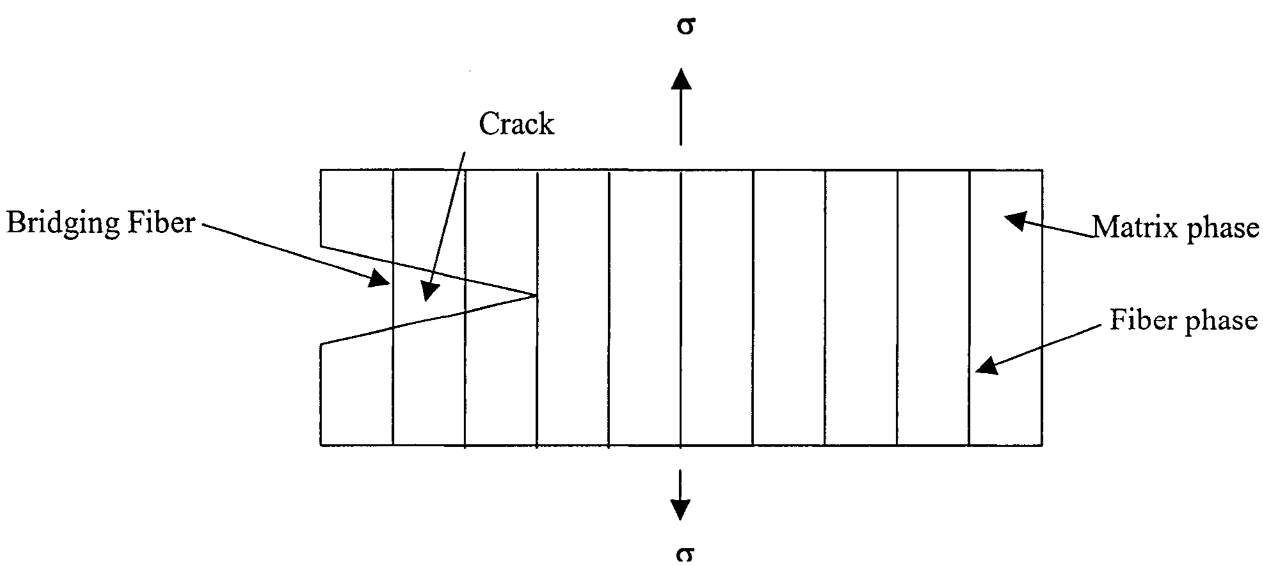


Figure 1.1. Fiber bridging mechanism in ceramic composites with a preexisting crack in the matrix

Mechanisms that do not involve failure by growth of a single crack have been observed. In that case, fracture toughness cannot be defined in the usual sense. Despite these complications, fracture mechanics can be applied to analyze failure of fiber composites, provided that the detailed mechanisms of failure are identified for each combination of the composite stress state. Such analysis provides insight into failure processes and allows definition of alternative material properties, which characterize the mechanical response. Furthermore, by relating these properties to microstructural parameters, the fracture mechanics analysis provides a means of designing optimum microstructures and anticipating microstructural changes in failure mechanism with changes in microstructural properties.

Section II

Paper I

"Simulation of Creep Crack Growth in Ceramic Composites"

Praveen Sodanapalli
Mechanical Engineering Department
University of Wyoming
Laramie, WY 82071

Dennis N. Coon
Mechanical Engineering Department
University of Wyoming
Laramie, WY 82071

Abstract

Tensile creep behavior and elevated temperature fatigue of fiber reinforced ceramic composites were investigated using Monte Carlo simulation. The simulated model consisted of a uniaxially loaded tow aligned the direction of the load. The simulation generated a Weibull distribution of fiber strengths, and Gaussian distributions of fiber modulus and radius. The simulation assumed a creep strain rate consisting of primary and steady state components each of which was modeled by a power law relationship. Power law exponents in the range of 0-7 for a selected SiC/SiC system at stress levels ranging from 60 MPa to 200 MPa were evaluated. A fatigue exponent of 3.03 ± 0.066 was predicted for nominal stress levels less than 150 GPa. The influence of initial crack length on the failure life times was also studied. Predicted failure response suggested a stress dependent creep process could be used to model experimental data and possibly evaluate the failure mechanism of reinforced test items.

Background

Ceramic matrix composites (CMCs) are candidate structural materials for high temperature applications due to their outstanding structural properties such as high specific strength, stiffness, and toughness. CMCs are candidates for applications for which lifetime at elevated temperature under stress is an important material characteristic. Owing to their recent development, however, only limited work has been published on the creep behavior of ceramic composites because experimental investigations of the elevated temperature creep behavior of fiber reinforced CMCs are costly and time consuming. There is a need to understand and predict the creep deformation behavior of CMCs as a function of basic properties of the constituents.

The progression of failure features in fiber-reinforced ceramics has been identified for monotonically loaded tensile loaded specimens [1]. Failure initiated with a single matrix crack growing in the direction perpendicular to the maximum principal stress. The fibers remained intact and bridge the crack in the matrix phase. Further load increase resulted in the formation of additional matrix cracks, and these matrix cracks were regularly spaced at about 400 μm in the direction of principal stress. Final failure of tension specimens was preceded by fiber pullout and then fiber failure. Growth of multiple cracks has also been observed [2]. Flexure failure mechanisms have been identified as matrix cracks that progress from the tension surface toward the compressive surface [2-4]. Final failure was attributed to buckling instability resulting from reduced cross section.

The cyclic-fatigue behavior of CMCs at high temperatures is not well understood. Elements such as environmental factors, creep of constituents, thermally induced stresses at interfaces, and interfacial sliding resistance can cause the reduction of fatigue life at high temperatures [5-7]. If the operating temperature is lower than that for the onset of creep of the fibers and the matrix, the decrease of sliding resistance due to relaxation of the residual stress is attributed to be the dominant fatigue mechanism [7]. When the temperature is high enough to produce strength degradation of fibers such as creep, the failure becomes complicated and is a priority for further research. Fatigue and creep damage mechanisms can operate simultaneously under high temperature cyclic loading Fatigue loading at high temperatures resulted in creep fatigue interactions which caused a reduction in the number of cycles to failure.

Limited lifetimes have been observed experimentally in ceramic composites at high temperatures [8-13], and several degradation mechanisms have been identified including creep, fiber/environment reaction, and wear of the fiber surfaces during cyclic loading. Creep has been identified as the predominant damage mechanism for fatigue at elevated temperatures [1-3,11,13,14]. Holmes investigated the elevated temperature fatigue response and modeled the creep strain rate using a power law, which is similar to the strain ratcheting law used in fatigue failures [15].

The purpose of the present paper is to discuss the results obtained from a Monte Carlo simulation of the tensile creep behavior of a unidirectional SiC/ SiC ceramic composite.

Monte Carlo Simulation

Monte Carlo simulation is the numerical solution of analytical models containing probabilistic characteristics. In Monte Carlo simulation, a computer performs the calculations of system behavior following well-defined mathematical relationships involving these probabilistic characteristics. In this manner, the solution is determined in part by mathematical relationships, and in part by the values of the various probabilistic characteristics. The validity of the numerical solution is determined by the appropriateness of the mathematical relationships and the probabilistic characteristics. Random number generators, often pseudo-random number generators, are used to determine the probabilistic characteristics. While any single solution will vary as the values of the probabilistic characteristics vary, performing the numerical solution many times can give insight into the behavior of the physical system. An analytical solution using variational calculus is an alternate approach to the solution to this class of problems. However, a Monte Carlo approach allows the unique individual responses to be observed while providing the average behavior of the analytical solution. This advantage makes Monte Carlo approaches very valuable.

The study of fiber and composite material fracture is reviewed and analyzed using statistical and probabilistic concepts. The justification for the use of statistical methods in the study of the brittle structures is well known [3,4,16-19]. The results obtained from strength tests, using identical specimens and identical laboratory conditions, are often observed to greatly vary about the mean value. The reason for the scatter observed in the strength of brittle materials is the inherent variability in the shape, size, orientation and the nature of molecular, microscopic and macroscopic defects, which are always present in all these materials. The distribution and density of the defect population within the specimen is likely to influence the type of statistical function describing the strength behavior.

A crucial task in the application of the Monte Carlo method is the generation of the appropriate random samples. In the computational practice of Monte Carlo methods, the required random numbers and random vectors are actually generated by the computer in a deterministic subroutine. In this case of deterministic generation, we speak of pseudo-random number and pseudo-random vectors. The statistics toolbox of the MATLAB™ was used to generate random number sets. Monte Carlo variables developed by the current simulation included fiber radii, fiber strength, fiber moduli, and force distribution exponents of individual fiber. The statistical parameters were user defined.

The success of a Monte Carlo calculation depends not only on the appropriateness of the underlying stochastic model but also, to a large extent, on how well the random numbers used in the computation simulate the random variables in the model. Chi-Square and Kolmogorov-Smirnov tests were used to check the randomness and the compatibility of these generated variable sets [20].

Modeling Analysis

The Monte Carlo model described in this communication simulated quasi-static crack growth due to creep of fibers in a rectangular tow that was loaded externally in tension. It was assumed that the fibers were uniformly positioned in a matrix. The sequence of steps used to simulate the above system is described in the following sections.

Fiber Tow

The material was assumed to be a rectangular fiber tow that consisted of unidirectional fibers impregnated in the matrix uniformly in 12 rows and 41 columns. The fibers were aligned in the direction of applied stress.

Matrix cracking originated from preexisting flaws, typified by a crack in the matrix with intact bridging fibers over its entire surface. The length of the existing crack was user defined in the current simulation and was set at 5 % of the tow length. This crack length would result in immediate failure of the monolithic material, but resulted in a stable crack in the composite material under lower levels of applied stress. The crack was assumed to run perpendicular to the applied stress. Crack stability was determined using the concepts of linear elastic fracture mechanics [21]. The applied stress intensity factor, K_I, was compared with the critical stress intensity factor K_{IC}. If K_{IC} was greater than K_I, the crack was stable. Otherwise the crack was extended to the next fiber column and checked for stability. This process of crack growth increased the number of bridging fibers by one entire column of fibers, and was repeated until crack stability was achieved.

Load Distribution

External load was distributed to the individual fibers using an isostrain assumption. This approach assumed that the individual fibers and the matrix were in a state of uniform strain while they could be experiencing non-uniform stresses.

The matrix that was in front of the crack tip was assumed to carry mechanical load that was of the same magnitude as the load carried by the fiber. This assumption was justified since both the matrix and the fiber modeled in this study were silicon carbide. In this model, the load acting behind the crack tip was carried entirely by the bridging fibers. This load distribution was justified since bridging fibers are observed to take most of the load behind the crack tip [21]. The stress in the individual fibers (σ_{fiber}) was calculated from the applied stress ($\sigma_{applied}$):

$$\sigma_{fiber} = \frac{\sigma_{applied}}{V_f} \tag{1}$$

where V_f = the fiber volume fraction.

Equation1 determined the load carried by fibers behind the crack tip. Since both fibers and the matrix were assumed to take the load in front of the crack tip, the load taken by the fibers in front of the crack tip was σ_{fiber} and strain in the fiber array (ε_{fiber}) was calculated using a simple uniaxial approximation:

$$\varepsilon_{fiber} = \frac{\sigma_{fiber}}{\overline{E}} \tag{2}$$

where $\overline{E}$ = average modulus of fiber array.

Combining Equations 1 and 2, the strain in the fiber array was determined from:

$$\varepsilon_{fiber} = \frac{\sigma_{applied}}{V_f \overline{E}} = \varepsilon_{i,j} \tag{3}$$

where $\varepsilon_{i,j}$ = strain in fiber i,j

i = horizontal position of fiber in the tow

j = vertical position of fiber in tow.

However, the strain in the fiber can also be expressed as:

$$\varepsilon_{i,j} = \frac{\sigma_{i,j}}{E_{i,j}} \tag{4}$$

where $\sigma_{i,j}$ = stress in the fiber i,j.

$E_{i,j}$ = modulus of the fiber i,j.

The stress in fiber $_{i,j}$ can also be calculated from:

$$\sigma_{i,j} = \left(\frac{\sigma_{applied}}{V_f}\right)\left(\frac{E_{i,j}}{\overline{E}}\right) \tag{5}$$

Notice that if all the fibers exhibit the same modulus, Equation 5 condenses to,

$$\sigma_{i,j} = \left(\frac{\sigma_{applied}}{V_f}\right) \tag{6}$$

Equation 6 suggests that all fibers would be under the same stress, and this condition corresponds to an iso-stress condition.

The amount of stress carried by bridging fibers was calculated by:

$$\sigma_{bfiber} = \frac{\sigma_{crack} a}{S} \tag{7}$$

where S = surface area of the crack,

a = area of individual fibers in the bridging zone,

σ_{crack} = sum of stresses carried by the fibers behind the crack tip.

The applied stress intensity factor K_I was calculated by:

$$K_I = Y\sigma\sqrt{C\pi} \tag{8}$$

where $\sigma = \sigma_{applied} - \sigma_{bfiber}$ (The applied stress on the tow),

C = the crack length,

Y = a constant ($\pi^{1/2}$ for an edge loaded surface crack).

Creep Strain Rate Modeling

The total creep rate, ε_{tow}, was expressed as the sum of the primary creep rate, ε_p, and the steady state creep rate, ε_s, as in [22]:

$$\varepsilon_{tow} = \varepsilon_p + \varepsilon_s \tag{9}$$

Here the primary creep was determined by the equation [22]:

$$\varepsilon_p = A\sigma^p t^m \tag{10}$$

where A = stress independent constant,

σ = stress acting in the fibers,

t =the time elapsed since the loading,

p ,m= exponent constants.

Negative values of m suggest decreasing primary creep rate with time.

Steady state creep strain rate was modeled as a power law [22]:

$$\varepsilon_s = B\sigma^n \tag{11}$$

where σ = stress in the fibers,

n= stress exponent,

B= a constant.

The physical effect of creep was modeled using an assumption of constant volume to result in a reduction in cross-sectional area [3]:

$$A_{i,j}^{new} = \frac{A_{i,j}^{prev}}{\varepsilon_{tow} + 1} \tag{12}$$

where $A_{i,j}^{new}$ = cross sectional area of the fiber i,j after creep,

$A_{i,j}^{prev}$ = Cross sectional area of the fiber i,j prior to creep.

The stress in the individual fibers after creep was then calculated from:

$$\sigma_{i,j}^{new} = \frac{F_{i,j}}{A_{i,j}^{new}} \tag{13}$$

where $\sigma_{i,j}^{prev}$ = stress in fiber i,j after creep.

Rules for Stress Redistribution

In this simulation, an inverse power law rule was used to model the distribution of force from a broken fiber to other fibers in the array:

$$\Delta F_{i,j} = F_{broken}\left[\frac{\left.1\middle/(d_{i,j})\right.^{n_D}}{\sum_i\sum_j\left(\left.1\middle/(d_{i,j})^{n_D}\right.\right)}\right] \tag{14}$$

where $\Delta F_{i,j}$ = force distributed to fiber i, j,

F_{broken} = force from broken fiber that is to be distributed to the other fibers,

$D_{i,j}$ = distance from fiber i,j to the broken fiber,

n_D = force distribution exponent.

Equation 14 can be summed over all fibers in that array to yield:

$$\sum_i\sum_j \Delta F_{i,j} = \sum_i\sum_j F_{broken}\left[\frac{\left.1\middle/(d_{i,j})\right.^{n_D}}{\sum_i\sum_j\left(\left.1\middle/(d_{i,j})^{n_D}\right.\right)}\right] = F_{broken} \tag{15}$$

Equation 15 indicates that force balance is maintained as a fiber is broken, and that the force on that fiber is distributed to the remaining fibers.

Creep Mechanism and Fiber Failure.

The simulation allowed all the fibers to creep under the load, and, consequently the radius of the fibers reduced as a result of creep. Each time fibers were allowed to creep was known as a timestep in the simulation. The reduction of fiber radius in each timestep resulted in increased stress on the fiber under a constant load. The stress increase was modeled using a constant force assumption [18] and a modification of Equation 13:

$$\sigma_{i,j}^{new} = \sigma_{i,j}^{prev} \left(\frac{r_{i,j}^{prev}}{r_{i,j}^{new}} \right)^2 \qquad \textbf{(16)}$$

where $\sigma_{i,j}^{prev}$ = stress on fiber i, j in the previous iteration step,

$r_{i,j}^{new}$ = radius of fiber i, j in the current iteration step,

$r_{i,j}^{prev}$ = radius of fiber i, j in the previous iteration step.

The stress on the fiber increased until the fiber stress reached the fiber strength. At this point the fiber broke. The load on the broken fiber was distributed to the other fibers using Equation 14. Then, crack stability was checked, creep was applied to individual fibers, and fibers stresses were again compared to fiber strengths. In this way the Monte Carlo simulation was iterated until complete failure of fiber array occurred. Inherent in the constant force assumption was no redistribution of forces occurred in the fiber array until a fiber broke.

Results

The parameters and the statistical distributions used in the Monte Carlo simulation are given in the Table I.

Fiber Characteristic	Distribution used	Value
Fiber Radius	Gaussian distribution	Mean=6.9 micron Std=1.3 micron
Fiber Strength	Weibull distribution	Mean=1.1 GPa m=3.6
Fiber Modulus	Gaussian distribution	Mean=145 GPa Std=30 GPa

Table 2.1. Monte Carlo variables of fiber characteristics used in this study.

Model parameter	Value
Initial crack length (a_o)	5% of tow length
Stress Intensity Factor (K_I)	4.0 MPa.m^1/2
Applied Stress (σ_{app})	110MPa
Fiber Volume (V_f)	40%
Array Size of the fibers (i X j)	12 x 41
Stress exponent (n)	2

Table 2.2. Mechanical parameters used in this simulation.

The initial application of the stress resulted in the pure mechanical failure of a small number of fibers. This mechanical failure was due to failure of weaker fibers generated from the strength distribution, and was consistent with previous numerical studies [23]. The simulation allowed all fibers to creep under load. The fibers in front of the crack carry smaller loads than the bridging fibers as some of the load in front of the crack is carried by the matrix phase. Accumulation of creep strain led to the failure of some bridging fibers, which led to the weakening of the bridging zone. This weakening of the bridging zone led to crack growth until the crack stabilized by formation of additional bridging fibers. This process was continued until all the fibers were broken.

It appeared that there were three distinct regions in the failure of a fiber tow: crack growth incubation, crack growth, and fiber domination. Crack growth incubation was an initial period where small numbers of failed bridging fibers resulted in crack stability in the absence of the formation of new bridging fibers by crack growth. The crack growth incubation period typically lasted for about 20% of the lifetime of the tow. This phase was followed by the crack growth phase in which the crack propagated through the matrix at a fairly constant rate giving way to the fiber domination phase. Fiber domination was characterized by complete matrix cracking and a short period of rapid fiber failure leading to failure of the material. The mechanism of failure is illustrated in Figure 2.1. The most important characteristics of the plot are the crack incubation region, the instability point (iteration step where the crack is unstable), crack propagation rate, fiber domination point (iteration step where the complete matrix is cracked and fibers dominate the system) and lifetime. Each of the above characteristics was studied by varying the standard parameters.

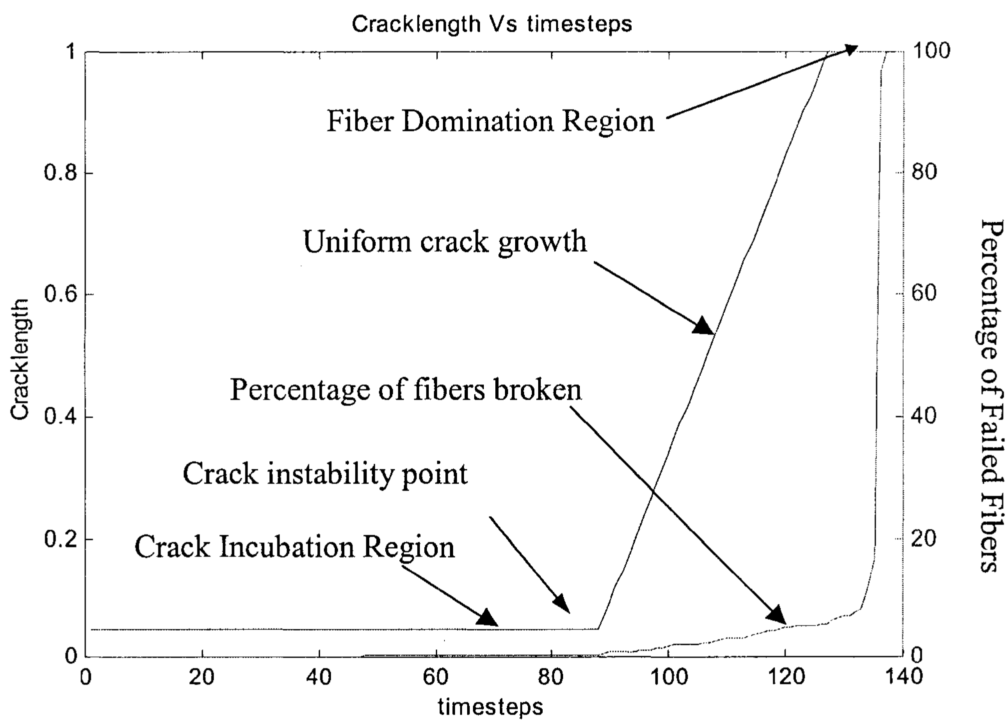


Figure 2.1. Crack Length (as a fraction of tow length) and percentage of fibers failed as a function of timesteps.

The total number of time steps indicates the total number of iteration steps completed for the failure of the whole fiber tow. In this way the number of timesteps to failure gives a relative measure of the lifetime of the fiber tow. As shown in Figure 2.1, the total number of timesteps taken for the complete failure of the fiber was 137. There was complete fiber domination beginning at timestep 126. From the Figure 2.1, we can observe that 90 % of fibers broke in the last 10 % of lifetime, which agrees well with the published data [8-13].

Convergence analysis

The convergence analysis of the Monte Carlo simulation was determined for up to 100 independent solutions of the simulation as shown in the Figure 2.2. This convergence analysis showed that the simulation yielded identical values of the mean and standard deviation when the number of solutions was above 35. Therefore, 50 runs were conducted for each variation of a parameter.

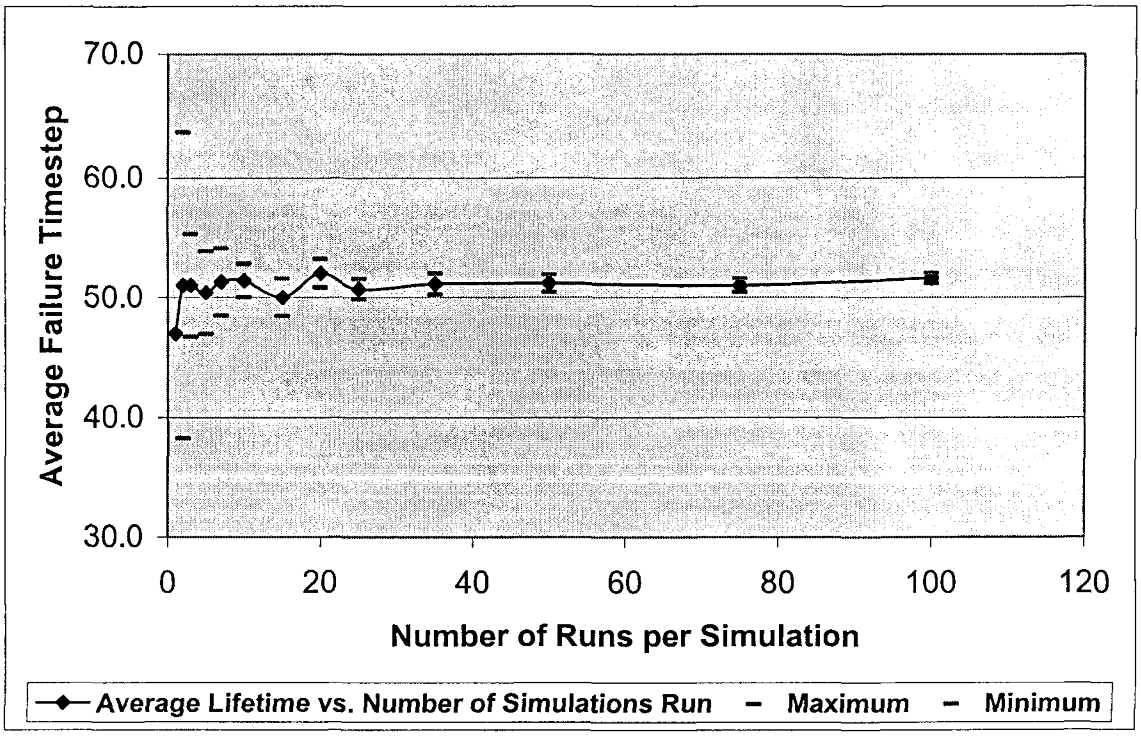


Figure 2.2. Convergence Analysis of the Monte Carlo simulation described in this communication.

Effect of Nominal Stress

The primary motive for this simulation was to examine the effects of applied load on the creep life of the fiber tow. The simulation was run for stress levels ranging from 60 MPa to 200 MPa. The plot of the applied nominal stress versus lifetime is shown in Figure 2.3. The plot clearly shows that the lifetime of the fiber tow decreased nonlinearly as the nominal stress increased. The mean of 50 simulations at each stress level was plotted with the error bars showing the standard deviation.

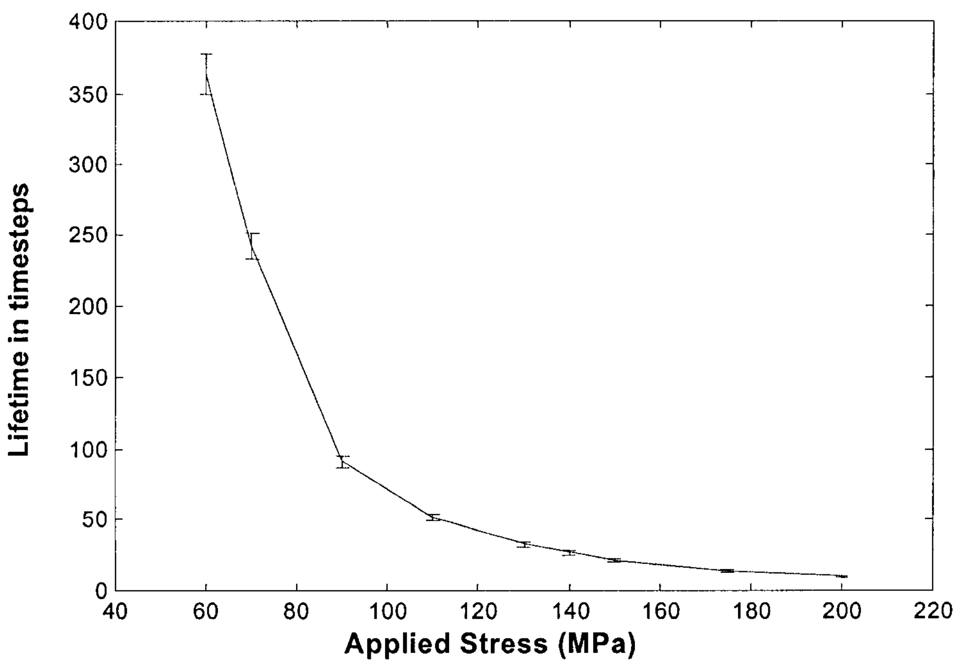


Figure 2.3. Plot of applied stress versus the failure lifetime.

Fatigue type behavior is defined as the nonlinear decrease with lifetime with increased stress, and is traditionally modeled with the following expression [16]:

$$t_f = A(\sigma_{nom})^{-n_f} \tag{17}$$

where t_f = lifetime,

A= constant,

σ_{nom} = nominal stress,

n_f = fatigue exponent.

Transformation of both the time and stress to a logarithmic scale yields linear behavior as shown in the Figure 2.4. Figure 2.4 suggests fatigue type behavior with a fatigue exponent of 3.03 ± 0.066. The fatigue exponent obtained from the simulation compares very well to the reported value of 2.92+0.034 [10-11].

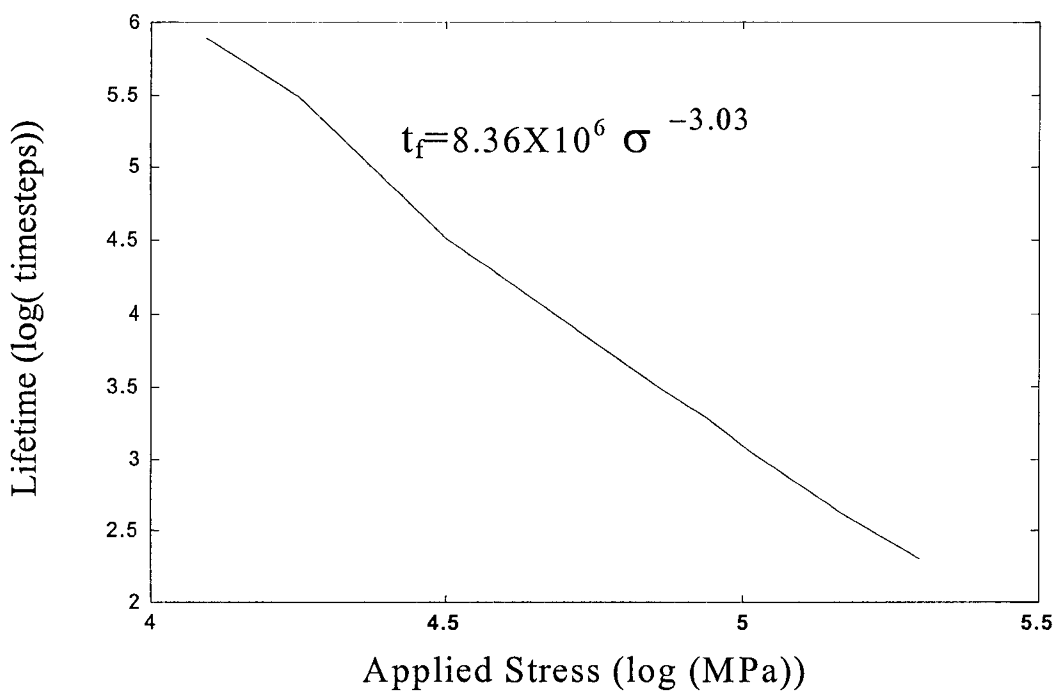


Figure 2.4. Fatigue behavior of ceramic fiber tow predicted using the simulation described in this communication.

It was observed that there was no crack propagation even in the absence of bridging fibers if the applied stress was less than 68.5 MPa. This lack of crack propagation was due to the Critical effective stress that resulted in crack instability in the absence of bridging fibers as given by:

$$\sigma_{crtical} = \frac{K_{Ic}}{Y\sqrt{\pi C}} \tag{18}$$

where $\sigma_{crtical}$ = Critical effective stress that can make the crack unstable,

K_{Ic} = fracture toughness of the composite,

C= initial crack length.

From Equation 18, it can be concluded that if the applied stresses are below the effective critical stress, then there will not be any crack growth in the composite. If all the bridging fibers are assumed to be intact, then the critical applied stress to cause the crack growth (at a crack length of 5% of the tow length) is approximately 82.9 MPa. The above results are in agreement with the proposition of infinite lifetime for low stresses, and are consistent with the literature [18].

Effect of Stress Exponent

The creep strain rate was modeled using a power law [Equation 10-12]. The influence of the stress exponent (n) on the failure lifetimes of the fiber tow was studied. The stress exponent was varied from 2 to 2.5 and the results are plotted in Figure 2.5.

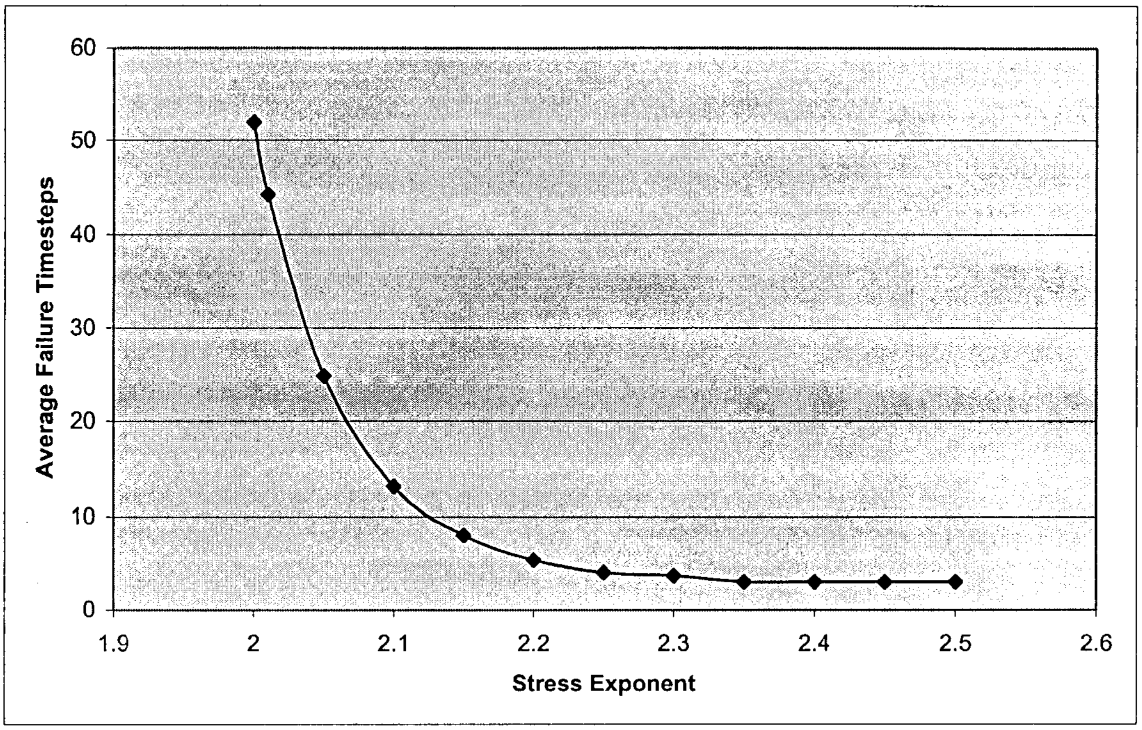


Figure 2.5. Influence of the stress exponent on the predicted lifetime

It can be observed from Figure 2.5 that the failure lifetime is very sensitive to the changes in the stress exponent. For stress exponent values that are greater than 2.25, fatal failure resulting in the fracture of all the fibers in one or two timesteps was observed. It was also observed that the matrix wais not fully cracked in the above cases, and the fiber bridging mechanism seem to have been dominated by the creep of the fibers.

Effect of initial crack length

Another important factor considered was the value of initial crack length and its effect on the creep lifetime of the fiber tow. A set of simulations were run in which the initial crack length is varied from 2.5% of tow length to 100 % of tow length (fully cracked matrix). Figure 2.6 shows the relationship between initial crack length and lifetime. Figure 2.6 shows that there is little functional dependence of initial crack length on lifetime beyond 5% of tow length. This observation shows that the lifetime of fiber tow depends largely on the fibers in a fiber-dominated system rather than on the initial crack length.

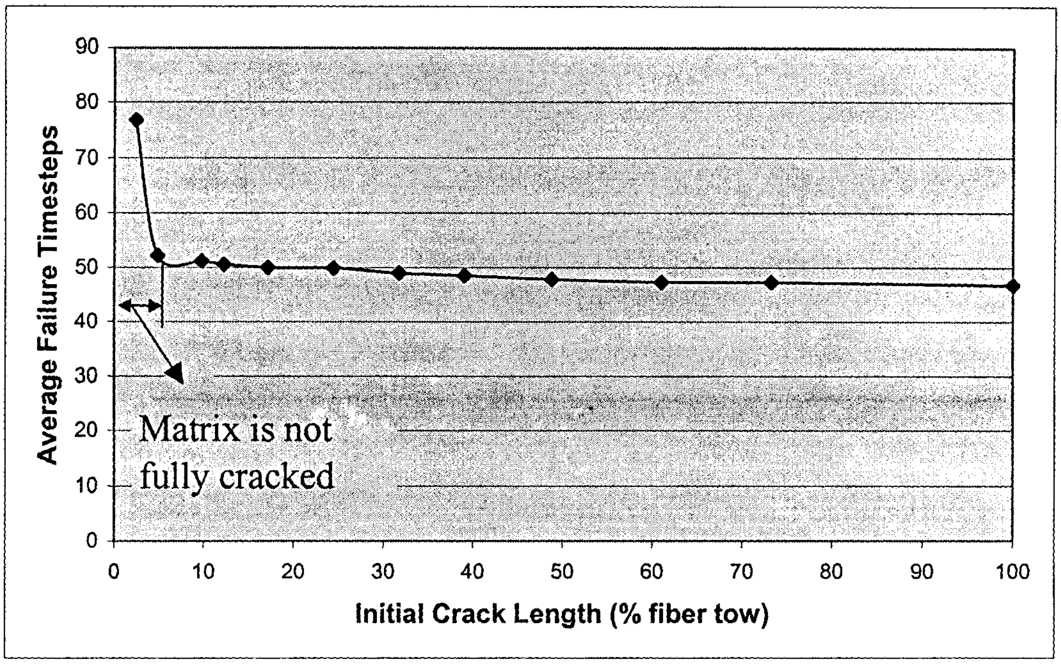


Figure 2.6. Effect of initial crack length on the predicted lifetime.

The initial crack length was not stable for lengths less than or equal to 5% of the fiber tow length. Therefore, the crack propagated and reached a specific point irrespective of its initial length. This observation can be explained as follows. From the fracture mechanics [24] the following equation can be written:

$$C_{crtical} = \frac{1}{\pi}\left[\frac{K_{Ic}}{Y\sigma_{effective}}\right]^2 \tag{19}$$

where $C_{crtical}$ = critical crack length below which crack is unstable,

$\sigma_{effective}$= effective stress acting in the fiber bridging zone,

K_{Ic}= fracture toughness of the composite.

From Equation 19, it may be observed that there is a critical crack length below which the crack is unstable for a given state of stress. This instability leads to crack growth till the crack becomes stable by fiber bridging mechanisms. Consequently, the lifetime of the fiber tow does not seem to change in the given range of initial crack length, i.e.

irrespective of the initial crack length, the crack stability mechanisms work in such a way that the failure times are independent of the initial crack length. The fiber domination point was reached faster as the initial crack length was increased.

To investigate this independence of crack length further, crack growth rate is plotted against initial crack length (Figure 2.7). From Figure 2.7 it appears that the crack rate decreased as the initial crack length increased. When the simulation was initiated with a large crack length, the bridging zone was large and the crack remained stable for a longer time. However this situation soon led to the fiber domination zone, where only fiber creep determined lifetime, which explains the observed behavior.

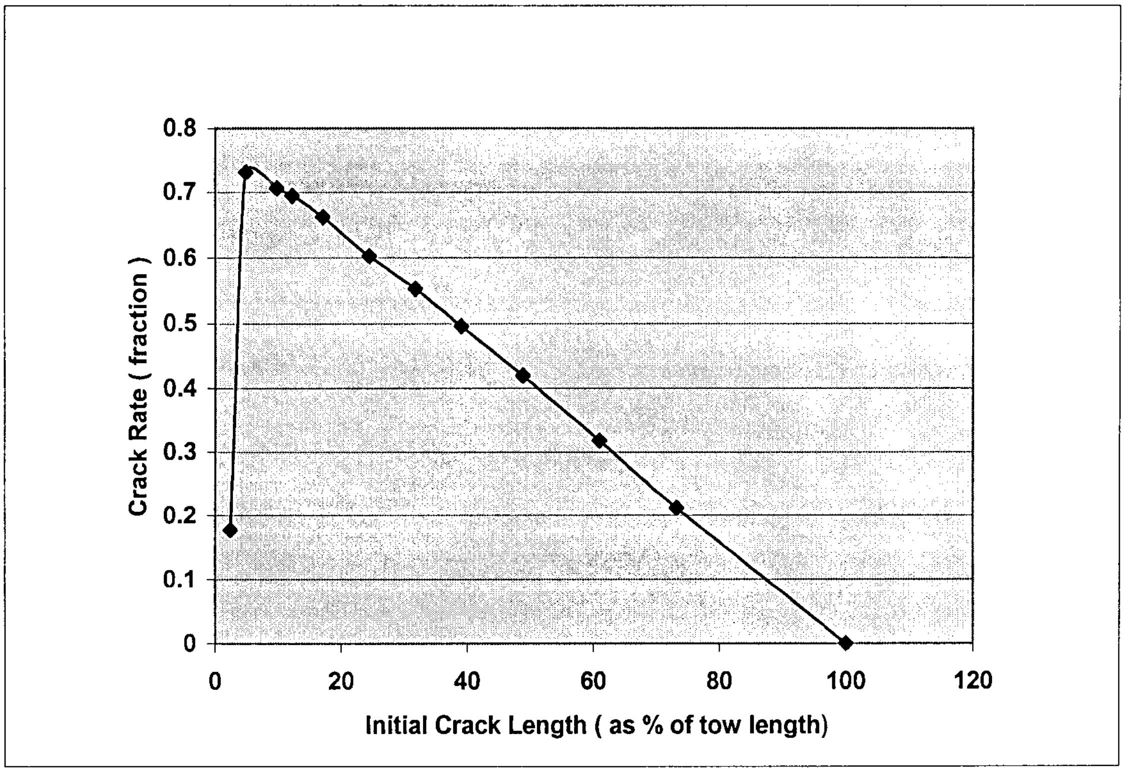


Figure 2.7. Crack growth rate as a function of initial crack length.

Conclusions

Tensile creep behavior and the elevated temperature fatigue of fiber reinforced ceramic composites were investigated using a Monte Carlo simulation. The simulated model consisted of a uniaxially loaded tow consisting of unidirectional fibers aligned in the direction of the load. The simulation assumed a creep strain rate, which can be approximated by a power law relationship. Stress (power law) exponents in the range of 2-2.5 for a selected SiC/SiC system at stress levels ranging from 60 MPa to 200 MPa were evaluated. It has been observed that the failure lifetimes of the composites are sensitive to the changes in the stress exponents. A fatigue exponent of 3.03 ± 0.066 was predicted for nominal stress levels less than 150 GPa. The initial crack length was predicted to have no effect on the failure life times. It was also observed that there was a critical initial crack length below which the crack is unstable. The above observation agrees well with fracture mechanics theory. Predicted failure response suggested a stress dependent creep process could be used to model experimental data and possibly evaluate the failure mechanism of reinforced test items.

References

1. MARSHALL, D. B., COX, B. N., Acta Metallurgica, 35[11], (1987), p. 2607.
2. BEGLEY, M. R. EVANS, A. G. MCMEEKING, R. M., Journal of the Mechanics and Physics of Solids, 43[5], (1995), p.727.
3. MACDONALD, F AND COON, D. N, under review by the Journal of Materials Science, 2000.
4. MACDONALD, F AND COON, D. N, accepted for publication in the Journal of Materials Science, 2000.
5. HEREDIOA, F. E., MCNULTY, J. C., ZOK, F. W., AND EVANS, A.G., Journal American Ceramic Society, 78[8], (1995), p. 2097.
6. MORRIS, W. L., COX, B. N., MARSHALL, D. B., INMAN, R. V., AND JAMES, M. R., Journal of American Ceramic Society, 77[2], (1994), p. 459.
7. REYNAUD, P., ROUBY, D., FANTOZZI, G., ABBE, F., AND PERES, P., Ceramic Transactions, 57, (1995), p. 95.
8. ZAWADA, L. P., BUTKUS, L.M., AND HARTMAN, G.A., Journal of American Ceramic Society, 74[11], (1991) p .2858.
9. RAGHURAMAN, S., STUBBINS, J.F., FERBER, M. K., AND WERESZCZAK, A. A., Journal of Nuclear Materials, 212-215, 91994) p. 840.
10. HEADINGER, M. H., GRAY, P., AND ROACH, D. H., Presented at the Composites and Advanced Structures Cocoa Beach Conference, January 1995.
11. VERILLI, M.J., CALAMINO, A.M, AND BREWER, D.N., presented at the Composites and Advanced Structures Cocoa Beach Conference, January 1995.
12. LIN, H.T., BECHER, P.F., MORE, K.L., TROTORELLI, P. F., AND LARA-CURZIO, E., Oak Ridge National Laboratory, 1996
13. SUN, E.Y, LIN, S.T, AND BRENNAN, J.J., Journal of American Ceramic Society, 80[3], (1996) p. 3065.
14. HOLMES, J. W., Journal of Materials Science, 26 (1991) p.1808.

15. DICARLO, J. A, Journal of Materials Science, 21[1], (1986) p. 217.

16. COON, D. N. AND CALOMINO, A. M., under review by Journal of Materials Science, 1999.

17. WAGNER, D. H., Application of Fracture Mechanics to Composite Materials, Elsevier Science Publishing Company Inc, Vol 6, (1989), p.39

18. COON, D. N. AND MOTKUR, A., Journal of Materials Science, **35**, (2000) p. 3207.

19. ECKEL, A. J. AND BRADT, R. C., Journal of American Ceramic Society, 72[3], (1989) p. 455.

20. LAPIN, L.L., Modern Engineering Statistics, Duxbury Press (1997).

21. MARSHALL, D. B. AND EVANS, A. G., Fracture Mechanics of Ceramics – Vol 7, (1986), p. 1.

22. PARK, Y.H. AND HOLMES, J. W., Journal of Materials Science, 27, (1992) p. 6341.

23. CHOI, S. R., SALEM, J. A., NEMETH, N. N., Journal of Materials Science. 33[5], (1998) p. 1325.

24. HERTZBERG, R. W., Deformation and Fracture Mechanics of Engineering Materials, John Wiley& Sons, Inc. (1989).

Section III

Paper II

"Simulation of the Effect of Fiber Characteristics on Creep Crack Growth in Ceramic Composites"

Praveen Sodanapalli
Mechanical Engineering Department
University of Wyoming
Laramie, WY 82071

Dennis N. Coon
Mechanical Engineering Department
University of Wyoming
Laramie, WY 82071

Abstract

Tensile creep behavior and the elevated temperature fatigue of fiber reinforced ceramic composites were investigated using a Monte Carlo simulation. The simulated model consisted of a uniaxially loaded tow aligned the direction of the load. The simulation assumed a creep strain rate consisting of primary and steady state components each of which could be approximated by a power law relationship. Average magnitude and standard deviation of fiber characteristics were varied in the simulation while keeping the creep model constant. Low values of fiber radius appeared to increase lifetimes while higher values had little impact on the failure lifetimes. An increase in the fracture toughness increased the lifetime of the composite for moderate values but had little effect for higher values of fracture toughness. Both characteristic fiber strength and the Weibull modulus were predicted to have significant effect on the creep life of the fiber tow. An increase in either the characteristic strength or Weibull modulus was predicted to result in an increase in creep life with the former having more influence than the latter. Due to the modeling assumptions, the elastic modulus of fibers was observed to have little impact on the creep life of the fiber tow.

Background

Ceramic matrix composites (CMCs) are the candidate structural materials for high temperature applications due to their outstanding structural properties such as high specific strength, stiffness and toughness. Very little is currently known about the influence of composite microstructure on creep behavior owing to their recent development, thus only limited work has been published on the creep behavior of ceramic composites. Because experimental investigations of the elevated temperature creep behavior of fiber reinforced CMCs are costly and time consuming there is a need to first understand and predict the creep deformation behavior of CMCs as a function of basic properties of the constituents.

Limited lifetimes have been observed experimentally in ceramic composites at high temperatures [1-6], and several degradation mechanisms have been identified including creep, fiber/environment reaction and wear of the fiber surfaces during cyclic loading. Creep has been identified as the predominant damage mechanism for fatigue [4,7-10] at elevated temperatures. Holmes investigated the elevated temperature fatigue response and modeled the creep strain rate using a power law, which is similar to the strain ratcheting law used in fatigue failures [8].

It has been reported that [9] prediction of the peak load requires statistical analysis of fiber fracture and pullout. However, fibers aligned with the principal stresses direction dictate the properties of a reinforced ceramic [9-11]. This observation implies that mechanical performance of fiber–reinforced ceramics can be improved by careful consideration of fiber properties and fiber architecture. The purpose of this paper is to discuss the effect of various fiber characteristics on the creep life of the fiber tow by simulating a unidirectional SiC/SiC ceramic composite.

Monte Carlo Simulation

Monte Carlo simulation is the numerical solution to the physical problems containing probabilistic characteristics. As compared to analytical solutions, the Monte Carlo approach allows the unique individual responses to be observed while providing the average behavior of the analytical solution. The validity of the numerical solution is determined by the appropriateness of the mathematical relationships and the probabilistic characteristics. A crucial task in the application of the Monte Carlo method is the generation of the appropriate random samples. The statistics toolbox of MATLAB™ package was used to generate random number sets. Monte Carlo variables developed by the current simulation included fiber radii, fiber strength, fiber moduli, and force distribution exponents of individual fiber. The success of a Monte Carlo calculation depends to a large extent, on how well the random numbers used in the computation simulate the random variables in the model. Chi-Square and Kolmogorov-Smirnov tests were used to check the randomness and the compatibility of these generated variable sets [12].

Modeling Analysis

The Monte Carlo model simulated quasi-static crack growth due to creep of fibers in a rectangular tow that is unidirectionally loaded [15]. The material was assumed to be a rectangular fiber tow that consisted of unidirectional fibers impregnated in the matrix uniformly in 12 rows and 41 columns. The fibers were aligned in the direction of applied stress.

The model simulates a preexisting crack with intact bridging fibers over its entire surface. The crack was set at 5 % of the tow length and it is assumed to run perpendicular to the applied stress. Crack growth was determined using the concepts of Linear Elastic Fracture Mechanics [13]. External Load was distributed to the individual fibers using an isostrain assumption. In this manner the stiffest fibers preferentially carry the highest proportion of the applied force. The matrix that was in front of the crack tip was assumed to carry mechanical load that was of the same magnitude as the load carried by the fiber while the load acting behind the crack tip is assumed to be taken entirely by the bridging fibers.

The total creep rate, ε_{tow}, was expressed as the sum of the primary creep rate, ε_p, and the steady state creep rate, ε_s, as [14]:

$$\varepsilon_{tow} = \varepsilon_p + \varepsilon_s \qquad (1)$$

Here the primary creep was determined by the equation:

$$\varepsilon_p = A\sigma^p t^m \tag{2}$$

where A = Stress independent constant

σ =The stress acting in the fibers

t =the time elapsed since the loading

p ,m= exponent constants.

Steady state creep strain rate was modeled as a power law [14]:

$$\varepsilon_s = B\sigma^n \tag{3}$$

where B = constant

n= stress exponent.

The simulation described in this communication allowed all the fibers to creep under the load, and, consequently the radius of the fibers reduced [15]. This led to an increased stress on the fiber under a constant load. The stress increase was modeled using a constant force assumption [9, 15]. The stress on the fiber increased until the fiber stress reached the fiber strength. At this point the fiber broke. An inverse power law rule was used to model the distribution of force from a broken fiber to other fibers in the array. The load on the broken fiber was distributed to the other fibers. Then, the crack stability was checked, creep was applied to individual fibers, and fiber stresses were again compared to fiber strengths. In this way the Monte Carlo simulation was iterated until the complete failure of fiber array occurs.

Results

The performance of fiber reinforced ceramic matrix composites can be improved by carefully choosing the fiber properties. In this study the effect of some of these properties on the lifetime of the fiber tow were investigated. The distribution of fiber radii was modeled as a Gaussian distribution about a mean of 6.9 microns and a standard deviation of 1.3 microns [16]. The distribution of fibers strengths was modeled as a Weibull distribution with a characteristic strength of 1.1 GPa and a Weibull parameter of 3.6 [17]. The distribution of fiber moduli was modeled as a Gaussian distribution about an average of 145 GPa and a standard deviation of 30 GPa [17]. The distribution of force distribution exponents was modeled as a uniform distribution with a range of ± 1 from the expected value (uniformly distributed from k-1 to k+1 for a force distribution exponent of k).

Selected simulation parameters were systematically varied to determine their influence on the simulated response. Those selected simulation parameters included fracture toughness of the composite, volume fraction of fiber, characteristic strength of the fiber, Weibull modulus for the fiber, and fiber moduli. Fifty runs per simulation were run to reduce the uncertainty in the typical results. The mean simulated lifetime was used to predict the effect of the fiber properties.

Effect of the radius of the fibers

The influence of the radius of the fibers on the lifetime of the fiber tow was investigated by running a series of simulations varying the radius of the fibers from 1 to 15 microns. The results are plotted on Figure 3.1. It can be noted here that the stress intensity factor (K_I) is a function of both crack length and the applied stress as in the equation:

$$K_I = \sigma_{effective} \sqrt{\pi C} \tag{4}$$

where K_I = Stress intensity factor

$\sigma_{effective}$ = Effective stress acting in the bridging zone

C= crack length.

Both the crack length and the effective stress acting in the bridging zone are functions of the radius of the fibers. For low radius values, the $\sigma_{effective}$ dominates Equation 4, and yields a low stress intensity factor results in a stable crack stable and higher predicted lifetimes. But as the fiber radius was increased beyond 5 microns, the crack length becomes the dominant factor in determining the stress intensity factor, prompting the crack to grow and fail quickly. However after a certain increase in the radius of the fibers this theory does not have any effect on the failure lifetimes. This situation resulted from the matrix being fully cracked in these cases and the creep process alone determined the failure lifetimes.

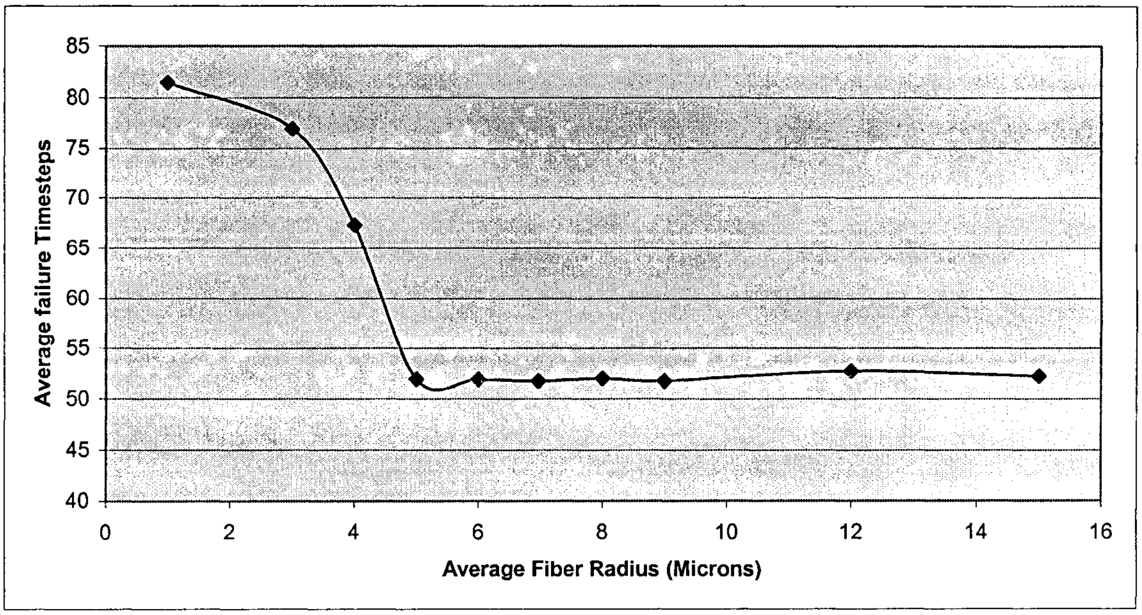


Figure 3.1. Effect of average fiber radius on the predicted lifetime.

Effect of fracture toughness

The simulation was used to predict lifetime for fracture toughness values ranging from 3 to 8 MPa•m$^{1/2}$, keeping all the other parameters constant. The fracture toughness determined the stability of the crack, and therefore, played an important role in the propagation of the crack. Figure 3.2 shows the predicted relationship between fracture toughness and creep lifetime. Failure lifetime increased with fracture toughness because the material had higher resistance to crack propagation. As the fracture toughness was increased in the fiber tow, the crack took longer to propagate across the fiber array. The simulation also predicted that the increase in the fracture toughness led to an increase in the crack incubation period and a decrease in the crack growth rate [15].

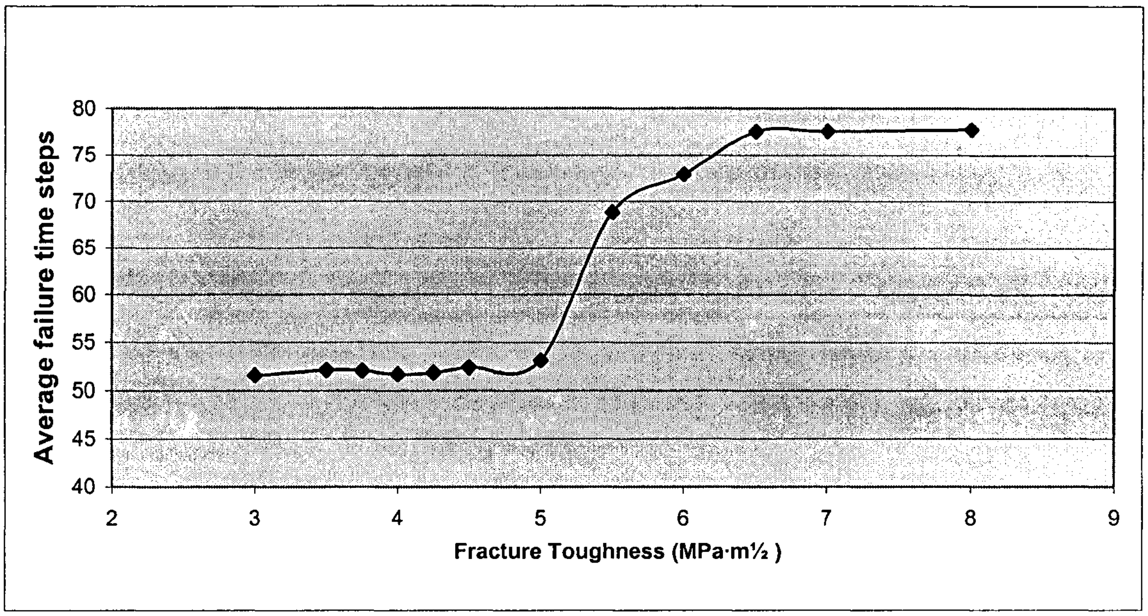


Figure 3.2. Effect of fracture toughness on the predicted lifetime.

Effect of Characteristic Fiber Strength

Characteristic strength is a measure of the average fiber strength, and is one of the parameters used to generate a Weibull distribution of fiber strengths. The effect of the characteristic fiber strength on the life of the fiber tow has been investigated. Figure 3.3 shows the plot of characteristic fiber strength as a function of failure lifetime. From the plot it can be concluded that as characteristic strength of the fibers was increased the failure lifetimes increase nonlinearly.

Weibull Modulus of Fiber Strength:

Weibull modulus is a parameter used to produce the Weibull distribution of fiber strengths and is inversely proportional to the width of the strength distribution. Varying the Weibull modulus yielded a similar nonlinear behavior (Figure 3.4) as the characteristic fiber strength. However it can be concluded from the equations that fit the behavior of both properties that the effect of Weibull modulus on failure times is less pronounced than that of characteristic fiber strength.

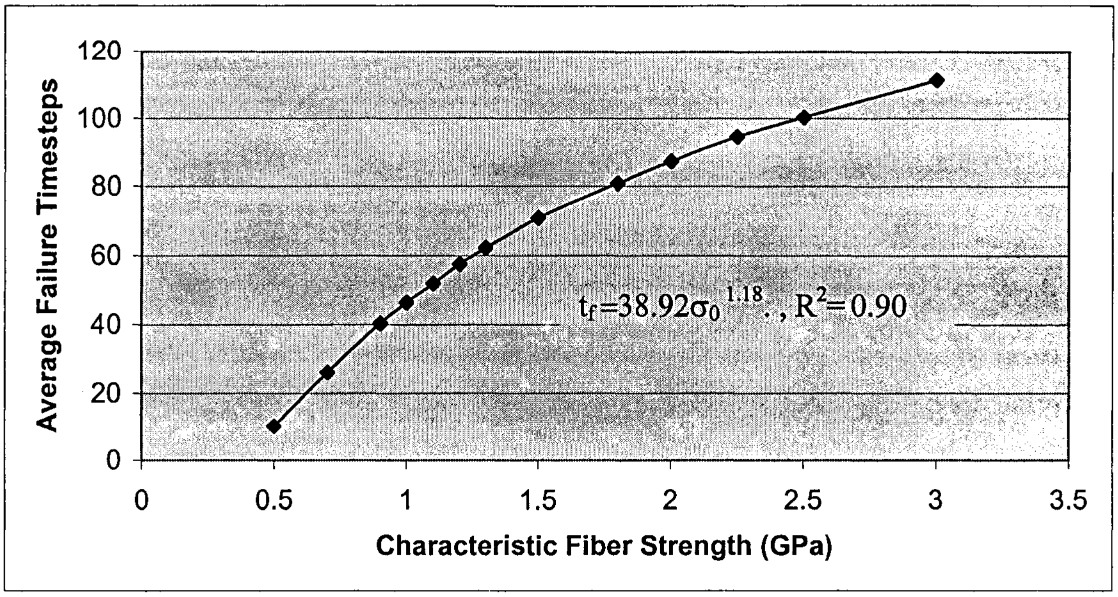


Figure 3.3. Effect of fiber characteristic trength on the predicted lifetime.

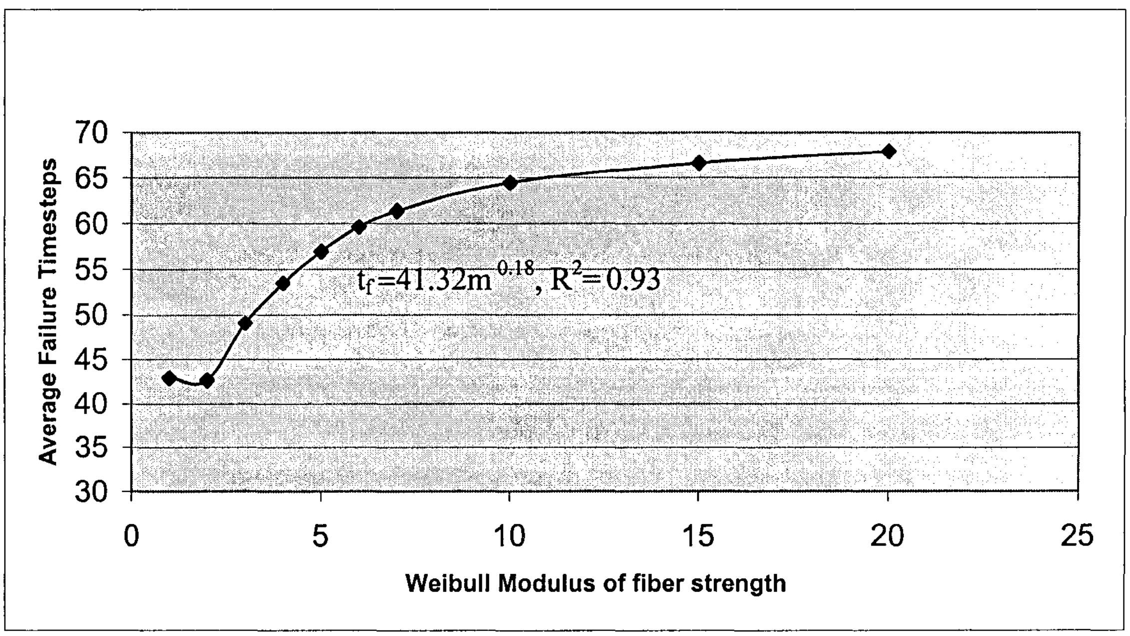


Figure 3.4. Effect of the Weibull modulus of fiber strength on predicted lifetime.

Effect of Volume Fraction of Fibers

The influence of the volume fraction of the composite on the failure lifetime was investigated using the Monte Carlo simulation. The simulation predicted an increase in the lifetime with the increase in the volume of the fibers present in the matrix (Figure 3.5). The fiber tow failed without even cracking fully for volume fractions that were less than 0.3. The fiber domination region did not exist for these cases. It was also noted that the crack did not grow from the initial crack in the systems having a fiber volume fractions greater than 0.6. It can also be noted from the Figure 3.5 that an increase in the fiber volume fraction beyond 0.7 had little impact on the failure lifetime of the fiber tow.

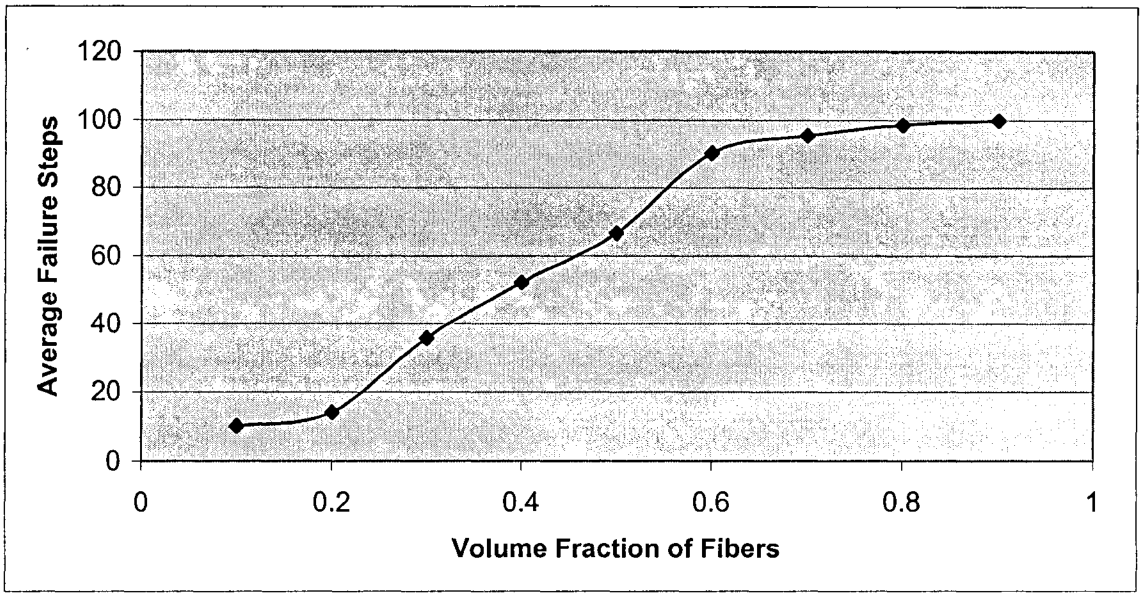


Figure 3.5. Effect of volume fraction of fibers on predicted lifetime.

Effect of Elastic Modulus of Fibers

The elastic moduli of fibers were generated from a normal distribution. The mean and standard deviation parameters of the normal distribution were varied to find the possible influence on the creep lifetime. The mean elastic modulus of the fibers was predicted to have no effect on the creep lifetime (Figure 3.6). The mean elastic modulus was used only in the determination of stresses in the individual fibers [15]:

$$\sigma_{i,j} = \left(\frac{\sigma_{applied}}{V_f} \right) \left(\frac{E_{i,j}}{\overline{E}} \right) \quad \textbf{(5)}$$

where $\sigma_{applied}$ =Applied stress on the fiber tow

$\sigma_{i,j}$ = stress in the fiber i,j

V_f = the fiber volume fraction

$\overline{E}$ = Average modulus of fiber array

$E_{i,j}$ = modulus of the fiber i,j

i = horizonatal position of fiber in tow

J = veritical position of fiber in tow.

Careful consideration of Equation 5 suggests the effect of elastic modulus from the distribution is cancelled by the mean Elastic modulus present in the equation. Therefore, the mean elastic modulus would be expected to have no effect on the creep lifetime.

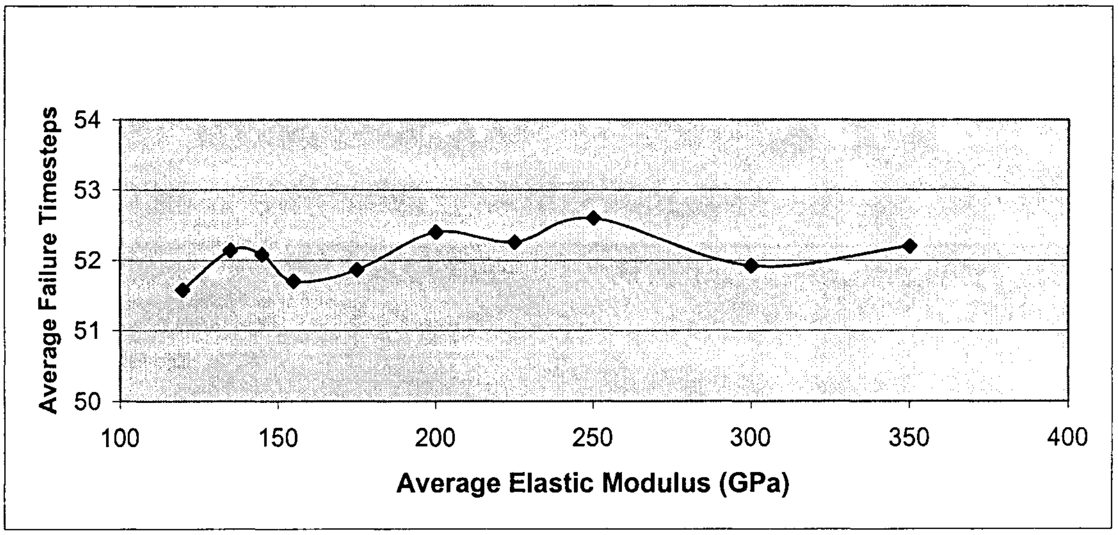


Figure 3.6. Effect of average elastic modulus of fibers on predicted lifetime.

However the standard deviation in the distribution of elastic modulus of the fibers appeared to have a mild effect on the lifetime of the tow. The results on the effect of the standard deviation of the elastic modus on the lifetime are plotted in Figure 3.7. The spread in the Elastic modulus of the individual fibers was increased as the standard deviation was increased. This increase in the standard deviation caused some of the fibers to be highly loaded leading to their early failure and lower lifetimes. It can be concluded that the minimization of the standard deviation of the elastic modulus of the fibers can increase failure lifetime.

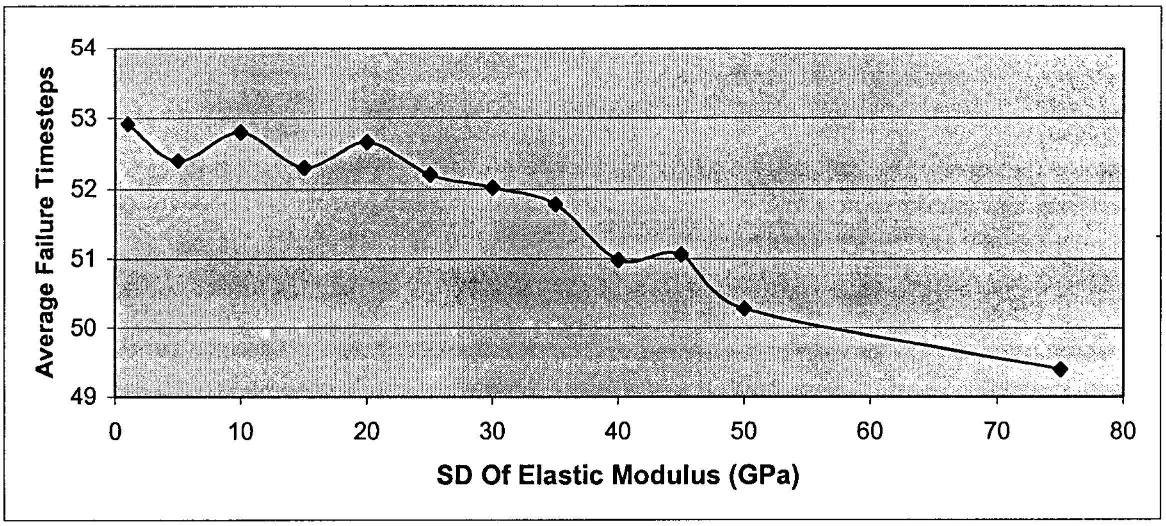


Figure 3.7. Effect of standard deviation of elastic modulus on predicted lifetime.

Conclusions

The influence of the fiber characteristics on the failure lifetimes of a SiC fiber tow was predicted using a Monte Carlo simulation. A creep model with a creep rate modeled as a power law with published values for the power law parameters was used. The simulation examined the effects fiber radii, elastic moduli, and volume fraction and fiber strength on the creep life of the fiber tow. An increase in the creep life is predicted for a decrease in the fiber radius below 4 microns. The fiber radius seemed to have little effect on the creep life for radius values that are more than 6 microns. The simulation also predicted an increase in the crack incubation period and decrease in the crack growth rate for an increase in the fracture toughness. An increase in the characteristic strength or the Weibull modulus of the strength of the fibers was expected to increase the creep life of the fiber tow with the former having more influence than the latter. Increasing the fiber content in the matrix led to an increase in the lifetime. It was also observed that controlling the fiber content in the tow could control the crack growth and crack incubation. Minimizing the spread in the values of the elastic modus of the fibers may lead to an increase in the lifetime of the fiber tow. The failure lifetime of a fiber reinforced ceramic matrix composite can be controlled by carefully choosing its fiber characteristics.

References

1. ZAWADA, L. P., BUTKUS, L.M., AND HARTMAN, G.A., Journal of American Ceramic Society, 74[11], (1991) p .2858.

2. RAGHURAMAN, S., STUBBINS, J.F., FERBER, M. K., AND WERESZCZAK, A. A., Journal of Nuclear Materials, 212-215, 91994) p. 840.

3. HEADINGER, M. H., GRAY, P., AND ROACH, D. H., Presented at the Composites and Advanced Structures Cocoa Beach Conference, January 1995.

4. VERILLI, M.J., CALAMINO, A.M, AND BREWER, D.N., presented at the Composites and Advanced Structures Cocoa Beach Conference, January 1995.

5. LIN, H.T., BECHER, P.F., MORE, K.L., TROTORELLI, P. F., AND LARA-CURZIO, E., Oak Ridge National Laboratory, 1996.

6. SUN, E.Y, LIN, S.T, AND BRENNAN, J.J., Journal of American Ceramic Society, 80[3], (1996) p. 3065.

7. HOLMES, J. W., Journal of Materials Science, 26 (1991) p.1808.

8. DICARLO, J. A, Journal of Materials Science, 21[1], (1986) p. 217.

9. MACDONALD, F AND COON, D. N, under review by the Journal of Materials Science, 2000.

10. BEGLEY, M. R. EVANS, A. G. MCMEEKING, R. M., Journal of the Mechanics and Physics of Solids, 43[5], (1995), p.727. .

11. MACDONALD, F AND COON, D. N, accepted for publication in the Journal of Materials Science, 2000.

12. LAPIN, L.L., Modern Engineering Statistics, Duxbury Press (1997).

13. MARSHALL, D. B. AND EVANS, A. G., Fracture Mechanics of Ceramics – Vol 7, (1986), p. 1.

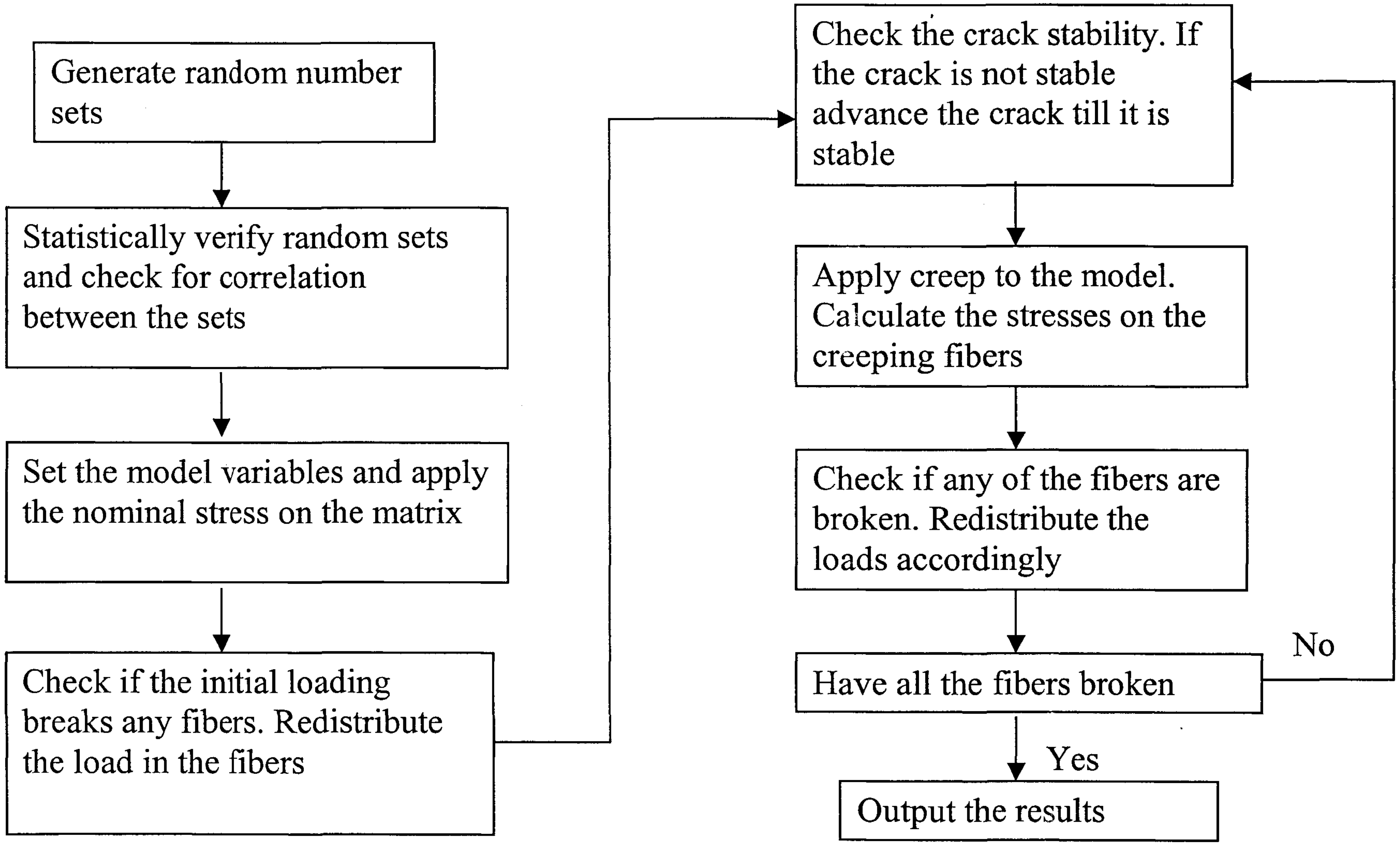
14. PARK, Y.H. AND HOLMES, J. W., Journal of Materials Science, 27, (1992) p. 6341.

15. SODANAPALLI, P. AND COON, D. N., to be submitted to Journal of Materials Science, 2000.

16. ECKEL, A. J. AND BRADT, R. C., Journal of American Ceramic Society, 72[3], (1989) p. 455.

17. SIMON, G. AND BUNSEL, A., Journal Materials Science, 19, (1984) p.3649.

Appendix A



Outline of the numerical simulation of creep crack growth in ceramic composites

Appendix B

```
% MAIN.m
% THIS IS THE MAIN PROGARAM FOR RUNNING THE SIMULATION.
%------------------------------------------------------------
% THE VARIABLES USED IN THIS PROGRAM ARE
% s= strength=strength of the fibers.
% c=modulus= modulus of fibers from rnd generator.
% d=rate= reaction rate
% e=fdist=force distribution exponents
% tstep=row matrix used for plotting timestep.
% Exp=mXn matrix which has 1 if the fiber is not broken and has zero if
that fiber is broken
% Exp is used in the percentbroc function in calculating the percentage
of broken fiber.
% perbroc=row matrix used for plotting the percentage of broken fiber
at each step.
% sigaap=the level of nominal stress here - assuming non-load bearing
matrix
% avgmod=Average modulus of the fibers
% whenbroke= It is the array which gives the timestep of failure of
each fiber.
% whenbroketemp=mxn matrix indicating the timestep at failure.
% sfiber is the array of the applied stress on the individual fibers
% tempradius=This is the (dynamic)radius of the fibers in the program
% r=radius= radius of the fibers from random generator
% =========================================================
%clear previously stored results
clear all
%=========================================================
% this part returns with the input variables.
global p Vf sigapp KIc Y avgmod m n appldstress
variables
%=========================================================
%variable parameters are set in RAND_GENE program
%you can also use the random distributions from the previous experiment
yesorno=input(' Do you want to use the random distribution from the
previous run:type(y/n)\n','s');
if yesorno=='y'
   VAR=wk1read('distribution');
   radius = VAR(:,1:41);
   strength = VAR(:,42:82);
   modulus=VAR(:,83:123);
   rate = VAR(:,124:164);
   fdist = VAR(:,165:205);
   tempradius = radius;
else
   RAND_GENE;
   %store the results from RAND_GENE in the appropriate arrays.
   radius = r;
   strength = s;
   modulus = c;
   rate = d;
   fdist = e;
   tempradius = r;
   % storing the results for use in later simulations.
   wk1write('distribution',[r s c d e]);
end
%========================================================
```

```
%Initialize an array to hold the timstep when each fiber broke
%the value of -1 is a flag to show that no timestep has been
%stored in the array position yet.
for i=1:m
   for j=1:n
      whenbroke(i,j)=-1;
   end
end
%Determine the column where the crack tip sets
q = round(p*n/100);
%print the initial crack position
fprintf('\nThe initial crack length is:%d\n',q);
% C is the crack tip line
% build a 12 X 1 matrix of 5 to identify the crack tip
C = ones(m,1).*5;
% represents the whole material matrix subjected to
% a userdefined crack tip
% MAT is   m X n+1  matrix
% sets the initial fiber state to 1 and adds the col of 5's
% to identify the crack tip
MAT = [ones(m,q),C,ones(m,n-q)];
% Now taking care of exposure matrix
% 1-fiber intact and 0- broben fiber
% Exp is a  m X n  matrix
% Exp array identifies all fibers as 1 (reacting or not?)
Exp = [ones(m,q),ones(m,n-q)];
% ==============assigning stress to fibers and matrix===============
% sfiber is the array of the applied stress on the individual fibers
% avgmod=Average modulus of the fibers is set as 145e9 Pa.
% sigapp=the level of nominal stress here - assuming non-load bearing
matrix
% It is assumed that matrix before the crack is non-load bearing and
% the matrix after the crack bears the same load as the fiber.
% Hence the following code.
% Note: The matrix creep and the subsequent matrix failure are not
taken in to account
% for the simplicity of the model. It is just made to take some load.
smatrix=appldstress;
for i = 1:m
   for j = 1:n
      if j>q
         sfiber(i,j) =(appldstress/(avgmod)).*(modulus(i,j));
      else
         sfiber(i,j) = (sigapp/(avgmod)).*(modulus(i,j));
      end
   end
end
%Determine which fibers break on initial mechanical loading
[sfiber,Exp,tempradius,whenbroketemp] =
FRACBROC_DIST(sfiber,strength,radius,tempradius,Exp,fdist,m,n,0);
%transfer the information from whenbroketemp to whenbroke
for i=1:m
   for j=1:n
      if(whenbroke(i,j)==-1)
         if(whenbroketemp(i,j)~=-1)
            whenbroke(i,j)=whenbroketemp(i,j);
         end
```

```
        end
    end
end
[p] = percentbroc(Exp,m,n);
%use a subroutine to determine the stability of crack tip
%the value of q return is the new position of the crack tip after the
crack is stable.
%the variable stable is a 1 or 0 depending on stability
[q,stable,sfiber]=CRACKSTABILITY(sigapp,sfiber,tempradius,KIc,Y,C,MAT,m
,n,q,smatrix,Vf);
fprintf('\nThe stabilized initial crack length is: %d\n',q);
%initalize the iteration counters
timestep = 1;
tstep(1) = timestep;
cracklen(1) = q;
perbroc(1)=p;
% Recording fiber status at each timestep in a file.
filename=strcat('profiles/matrixprofile',num2str(timestep));
wk1write(filename,Exp);
%=======================================================================
%Code will iterate until all fibers have broken
while(p<100.)
    %increment the counter, timestep
    timestep = timestep + 1;
    %determine if any fibers broke during this timestep
    [sfiber,Exp,tempradius,whenbroketemp] =
FRACBROC_DIST(sfiber,strength,radius,tempradius,Exp,fdist,m,n,timestep;
    %transfer the info from whenbroketemp to whenbroke
    for i=1:m
        for j=1:n
            if(whenbroke(i,j)==-1)
                if(whenbroketemp(i,j)~=-1)
                    whenbroke(i,j)=whenbroketemp(i,j);
                end
            end
        end
    end
    %check stability of the crack-tip
    [q,stable,sfiber] =
CRACKSTABILITY(sigapp,sfiber,tempradius,KIc,Y,C,MAT,m,n,q,smatrix,Vf);
    %-----------------------------------------------------------------
    % This part will apply the creep model to the fibers.
    % The result of this creeping would result in straining
    % by which the radius of the fibers are reduced ehnce more stress in
ther fibers.
    [sfiber,radius,tempradius]
=creep(sfiber,radius,tempradius,m,n,q,avgmod,timestep);
    % We can also combine our creep model with envirnment reaction model
by excuting the next step.
    % [sfiber,radius,tempradius] =
envreact(sfiber,radius,tempradius,rate,m,n,q);
    %-----------------------------------------------------------------
    %calculate the percent of fibers that have broken
    [p] = percentbroc(Exp,m,n);
    %store the time in an array for plottingtimestep = timestep + 1;
    tstep(timestep) = timestep;
    %store the crack length in an array for plotting
```

```
   cracklen(timestep) = q;
   %store the percant broken fibers in an array for plotting
   perbroc(timestep) = p;
   %--------------------------------------------------------------------
   % This part is intended to animate the process of fiber failure.
   % Recording the fiber status at each timestep.
   filename=strcat('profiles/matrixprofile',num2str(timestep));
   wk1write(filename,Exp);
end
%
========================================================================
%calculate the crach growth rate
crackrate=-1;
for i=1:timestep
   if(cracklen(i)==n)
      if(crackrate==-1)
         crackrate=(cracklen(i)-cracklen(1))./i;
      end
   end
end
% This will calculate crack length in case the tow fails before being
fully cracked
if(crackrate== -1)
   crackrate=(cracklen(timestep)-cracklen(1))./timestep;
end
fprintf('\nCrackrate: %f\n',crackrate);
%==============================================
% This is the animation part of the code.
figure(2);
z=1;
while z<=timestep    % reading from the files the status of fibers at
each timestep.
   y=linspace(1,12,200);
   plot(cracklen(z),y,'b-')
   hold on
   filename=strcat('profiles/matrixprofile',num2str(z));
   frame=wk1read(filename);
   % Converting the information into useful design.
   spy(frame);
   figfilename=strcat('fig/profile',num2str(z),'.jpg');
   %saveas(2,figfilename);
   M(:,z)=getframe(2);%record the movie
   z=z+1;
   hold off
end
%=====================================================
%convert the crack length (stored as colun number)
%into a fraction of the array length
cracklen1=cracklen/n;
%Now plot the results
figure(3),
plotyy(tstep,cracklen1,tstep,perbroc)
xlabel('timesteps');
ylabel('Cracklength');
title('Cracklength Vs timesteps');
fprintf('\nNumber of time steps before Failure: %d\n',timestep);
```

```
%*******************************************************************
%RAND_GENE.m
% Variation of input parameters appears to be accomplished by adjusting
the
% parameter in the random number generator statements.
%==================================================================
global m n avgrad avgmod weibstrength stdmod charstrength
% SET THE INPUT PARAMERS FOR THE RANDOM DISTRIBUTION GENERATION HERE.
%----------------------------------------------------------------------
% Average radius for the radius distribution is set in variables
function.
% Standard deviation(radius) = 1.3 microns.
stdrad=1.3e-06;
% characteristic fiber strength = 1.1 GPa and Weibull Mod = 3.6
%charstrength=1.1e+09;%IT IS FIXED IN VARIABLES PROGRAM
%weibstrength=3.6;%IT IS FIXED IN VARIABLES PROGRAM
% average modulus = 145 GPa and standard deviation = 30 GPa
%avgmod=145e09; %IT IS FIXED IN VARIABLES PROGRAM
%stdmod=30e09;%IT IS FIXED IN VARIABLES PROGRAM
% min expected reaction rate=- 10% and max expected rate=+ 10%
minrate=0.9;
maxrate=1.1;
% force distribution exponent n = 2
% min expected=-1 and max expected=+1
minexpnt=1;
maxexpnt=3;
%======================================================================
% The following set of loops is large to permit regeneration as
necessary
% Generation of  fiber radii
COUNT11 = 0;
while (COUNT11 < 100)
   rad = normrnd(avgrad,stdrad,m,n);
   rad1 = rad;
   probch1 = chi_norm(rad1);%this function perfoms the chi-norm test
and report about randomness of radii
   probks1 = kstest(rad1,'norm1');%this function perfoms the k-test
test and report about randomness of radii
   if ((probch1 > 0.05) & (probks1 > 0.05))
      %store passing results in array a
      r = rad;
      break;
   end
   COUNT11 = COUNT11 + 1;
   if COUNT11==99
      r=rad;
   end

end

% Generation of fiber strengths
COUNT22 = 0;
while (COUNT22 < 100)
   % note transformation from strength parameters to stat parameters
   str = weibrnd((( charstrength)^(-weibstrength)),weibstrength,m,n);
   str1 = str;
```

```
   probch2 = chi_norm(str1);%this function performs the chi-norm test
and report about randomness of strength
   probks2 = kstest(str1,'weib');%this function performs the k-test
test and report about randomness of radii
   cor1 = cortest(r,str1); %this function performs the cortest test and
report about compatibility? of radii and strength
   % 95% confidence is taken
   if ((probch1 > 0.05) & (probks1 > 0.05) & (cor1 > 0.05))
      % store passing results in array b
      s = str;
      break;
   end
   COUNT22 = COUNT22 + 1;
   if COUNT22==99
     s = str;
   end

end
% Generation of fiber moduli
COUNT33 = 0;
while (COUNT33 < 100)
   mod = normrnd(avgmod,stdmod,m,n);
   mod1 = mod;
   probch3 = chi_norm(mod1);
   probks3 = kstest(mod1,'norm2');
   cor2 = cortest(s,mod1);
   cor3 = cortest(mod1,r);
   if ((probch3>0.05) & (probks3>0.05) & (cor2>0.05) & (cor3>0.05))
      % store passing results in array c
      c = mod;
      break;
   end
   COUNT33 = COUNT33 + 1;
   if COUNT33 ==99
      fprintf('Normal distribution cannot be made');
      c=mod;
      end
end
% Generation of reaction rates of the individual fibers
COUNT44 = 0;
while (COUNT44 < 100)
   rat = unifrnd( minrate,maxrate,m,n);
   rat1 = rat;
   probch4 = chi_unif(rat1);
   probks4 = kstest(rat1,'unif1');
   if ( (probch4>0.05) & (probks4>0.05) )
      % store passing results in array d
      d = rat;
      break;
   end
   COUNT44 = COUNT44 + 1;
end
% Generation of force distribution exponents of the individual fibers
COUNT55 = 0;
while (COUNT55 < 100)
   dis = unifrnd(minexpnt,maxexpnt,m,n);
   dis1 = dis;
```

```
    probch5 = chi_unif(dis1);
    probks5 = kstest(dis1,'unif2');
    if ( (probch5>0.05) & (probks5>0.05) )
       % store passing results in array e
       e = dis;
       break;
    end
    COUNT55 = COUNT55 + 1;
end
```

```
%***********************************************************
% variables.m
% YOU CAN INPUT THE VARIABLES FOR THE ENTIRE SIMULATION HERE.
global p Vf sigapp KIc Y avgmod m n matstrength appldstress convfctr
avgrad stdmod charstrength weibstrength strsexponent
%-----------------------------------------------------------
%The array size is set here (n = num. of columns; m = num. of rows)
m = 12;
n = 41;
%-----------------------------------------------------------
%Set the crack length as a percent of the number of columns
% Default value of p is 5 %
p = 5;%input('what percentage of the total length');
%-----------------------------------------------------------
%Vf is the volume fraction equal to 40%
Vf = 0.4;
%-----------------------------------------------------------
% The level of nominal stress
% Set at a default value of appldstress=1.1e+08
appldstress=1.1e+08;

% Load carried by each fiber assuming non-load bearing matrix
sigapp = (appldstress/Vf);
%-----------------------------------------------------------
% The fracture toughness.
% Default value is KIc = (4e+06)
KIc = (4e+06);
% The crack shape parameter.
Y = pi^0.5;
%-----------------------------------------------------------
% average modulus and standard deviation of the fibers.
% Default values are avgmod=145e9 stdmod=30e09;
avgmod=145e9;
stdmod=30e09;
%-----------------------------------------------------------
% compensation factor from real time to timesteps
convfctr=5e6; % i.e convfctr*1sec=1 timestep set at 5e6
%-----------------------------------------------------------
% Strength of the matrix.
% Default value is matstrength= 145e9
matstrength= 145e9;
%-----------------------------------------------------------
% average radius of the fibers.
% This will be used by RAND_GENE,CRACKSTABILITY functions.
% Default value is avgrad=6.9e-06;
avgrad=6.9e-06;
%-----------------------------------------------------------
% characteristic fiber strength = 1.1 GPa and Weibull Mod = 3.6
charstrength=1.1e+09;
weibstrength=3.6;
%-----------------------------------------------------------
% Stress Exponent in the power law used to calculate steady state creep
rate
% Default value of strsexponent=n=2
strsexponent=2;
%-------------------------------------------------------------
```

```
%**************************************************
% chi_norm.m
function prob = chi_norm(rad1)
%=====================================================
%This function performs a chi-sqaured test of significance in
%comparison of a set of numbers to an expected uniform distribution
%======================================================

%The following variables must be passed to this function
%
%     x =  array that contains the data to be compared to a
distribution
%     nbins = the number of bins to divided the data into
%
%The following variables will be returned from this function
%
%     chisq = the chi-squared statistic
%     prob = the probability of significance
%===================================================
%Detemine the number of data points
[a,b]=size(rad1);
% i think a,b are used twicw or thrice this may be changed for good
%.............................................................
numdata = a*b;
%Sort the matrix into a single column matrix
x = [];
for i=1:b,
   x = [x;rad1(:,i)];
end
%number of bins the data is being divided
nbins = 10;
%Determine the maximum data point in the data set
maxdata=max(x);
%Determine the minimum data point in the data set
mindata=min(x);
%Devlop and array that defines the bin maximums
interval = (maxdata-mindata)/nbins;
for i = 1:nbins,
   binmax(i)=mindata+i*interval;
end

%Count the number of points in each bin
for a=1:nbins,
   datacount(a)=0;
   normbin(a)=0;
end

for i=1:numdata,
   for j=1:nbins,
      if j==1
         if x(i)<=binmax(j)
            datacount(j)=datacount(j)+1;
         end
      else
         if x(i)<=binmax(j)
            if x(i)>binmax(j-1)
               datacount(j)=datacount(j)+1;
```

```
            end
         end
      end
   end
end
%Develop a Guassian approximation of the expected distribution
normamount=normcdf(binmax,mean(x),std(x));
%count the expected number in each bin for a normal distribution
normbin(1)=numdata.*(normamount(1));
for i=2:nbins,
   normbin(i)=numdata.*(normamount(i)-normamount(i-1));
end
%Devlop an array that holds the incremental chi-sqaure sums
chiterms=((datacount-(normbin)).^2)./(normbin);
%Now sum to get the chi-square statistic
chisq=sum(chiterms);
%Determine the signicance of the above chi-square term
prob=chi2cdf(chisq,nbins-1);
%hist(datacount,10);
%xlabel('Magnitude of Observations');
%ylabel('Number of Observations');
```

```
%*************************************************
%chi_unif.m
%=====================================================
%This function performs a chi-sqaured test of significance in
%comparison of a set of numbers to an expected uniform distribution
%======================================================
function probch = chi_unif(x)
%The following variables must be passed to this function
%
%     x = array that contains the data to be compared to a distribution
%     nbins = the number of bins to divided the data into
%
%The following variables will be returned from this function
%
%     chisq = the chi-squared statistic
%     prob = the probability of significance
%===================================================
%Detemine the number of data points
nbins = 10;
[a,b]=size(x);
numdata = a*b;
%Determine the maximum data point in the data set
maxdata=max(max(x));
%Determine the minimum data point in the data set
mindata=min(min(x));
%Devlop and array that defines the bin maximums
interval=(maxdata-mindata)/nbins;
for i=1:nbins,
   binmax(i)=mindata+i*interval;
end
%Count the number of points in each bin
for a = 1:nbins,
   datacount(a)=0;
end

for i=1:numdata,
   for j=1:nbins,
      if j==1
         if x(i)<=binmax(j)
            datacount(j)=datacount(j)+1;
         end
      else
         if x(i)<=binmax(j)
            if x(i)>binmax(j-1)
               datacount(j)=datacount(j)+1;
            end
         end
      end
   end
end


%Develop an array that holds the incremental chi-square sums
chiterms=((datacount-(numdata./nbins)).^2)./(numdata./nbins);

%Now sum to get the chi-square statistic
chisq=sum(chiterms);
```

```
%Determine the significance of the above chi-square term
probch=chi2cdf(chisq,nbins-1);
%hist(datacount);
```

```
%**************************************************
%chi_weib.m
%================================================
%This function performs a chi-sqaured test of significance in
%comparison of a set of numbers to an expected uniform distribution
%================================================
function prob = chi_weib(y)
%The following variables must be passed to this function
%
%     x =  single column array that contains the data to be compared to
a distribution
%     nbins = the number of bins to divided the data into
%
%The following variables will be returned from this function
%
%     chisq = the chi-squared statistic
%     prob = the probability of significance
%================================================

%Detemine the number of data points
[a,b]=size(y);
numdata = a*b;

%Sort the matrix into a single column matrix
x = [];
for i=1:b,
   x = [x;y(:,i)];
end

%number of bins the data is being divided
nbins = 10;

%Determine the maximum data point in the data set
maxdata=max(x);

%Determine the minimum data point in the data set
mindata=min(x);

%Devlop and array that defines the bin maximums
interval=(maxdata-mindata)/nbins;
for i=1:nbins,
   binmax(i)=mindata+i*interval;
end

%Count the number of points in each bin
for a=1:nbins,
   datacount(a)=0;
   weibbin(a)=0;
end
for i=1:numdata,
   for j=1:nbins,
      if j==1
         if x(i)<=binmax(j)
            datacount(j)=datacount(j)+1;
         end
```

```
        else
           if x(i)<=binmax(j)
              if x(i)>binmax(j-1)
                 datacount(j)=datacount(j)+1;
              end
           end
        end
     end
  end

  %Develop a Weibull approximation of the expected distribution
  weibamount=weibcdf(binmax,1.1e+09^3.6,3.6);

  %Count the expected number in each bin for a Weibull distribution
  weibbin(1)=numdata.*(weibamount(1));
  for i=2:nbins,
     weibbin(i)=numdata.*(weibamount(i)-weibamount(i-1));
  end

  %Develop an array that holds the incremental chi-square sums
  chiterms=((datacount-(weibbin)).^2)./(weibbin);

  %Now sum to get the chi-square statistic
  chisq=sum(chiterms);

  %Determine the significance of the above chi-square term
  prob=chi2cdf(chisq,nbins-1);
```

```
%*********************************************
%cortest.m
function probco = cortest(x,y)
% Two sets of random numbers
[a b] = size(x);
n = a*b;

% Calculation of correlation coefficient
xd = mean(mean(x));
yd = mean(mean(y));
xi = x - xd;
yi = y - yd;
num = sum(sum(xi.*yi));
den = sqrt(sum(sum(xi.^2))) * sqrt(sum(sum(yi.^2)));
coef = num/den;

% Fisher's transformation
z = 0.5*log((1+coef)/(1-coef));

% Find the significance level
probco = erfc(abs(z*sqrt(n-3))/sqrt(2));
```

```
%**************************************
%kstest.m
function probks = kstest(M,name)
%Generate the random numbers
[m,n] = size(M);
numdata = m*n;

%The random numbers generated above are arranged in a single column
%matrix b. This makes easy to sort the whole array of random numbers.
b = [];
for i=1:n,
   b = [b;M(:,i)];
end

%sorting of random numbers in ascending order
x = sort(b);

%calculate the cumulative distribution of the sorted array
if strcmp(name,'weib') | strcmp(name,'weibull')
   y = weibcdf(x,1.1e+09^3.6,3.6);
elseif strcmp(name,'norm1') | strcmp(name,'normal1')
   y = normcdf(x,6.9e-06,1.3e-06);
elseif strcmp(name,'norm2') | strcmp(name,'normal2')
   y = normcdf(x,145e+09,30e+09);
elseif strcmp(name,'unif1') | strcmp(name,'uniform')
   y = unifcdf(x,.9,1.1);
elseif strcmp(name,'unif2') | strcmp(name,'uniform')
   y = unifcdf(x,1,3);
else
   error('This function doesnot support this distribution');
end

%z is the matrix that has probability of occurence of each random
%number generated
z1 = [];
for i=1:numdata,
   z1(i) = i/numdata;
end

z = z1';

%Calculation of the K-S statistic "D"
d = abs(z-y);
D = max(d);

%Finding the probability "Qks"
en = sqrt(numdata);
alam = (en + 0.12 + (0.11/en))*D;

for j = 1:2:100,
   Qks1(j) = 2*exp(-2*(j^2)*(alam^2));
end

for k = 2:2:100,
   Qks2(k) = -2*exp(-2*(k^2)*(alam^2));
end
```

```
probks = sum(Qks2) + sum(Qks1);
%plot(x,y,x,z);
%ylabel('Cumulative probability distribution');
%xlabel('x');
```

```
%*******************************************************************
%CRACKSTABILITY.m
% Function CRACKSTABILITY is used to check the crack stability
function [q,stable,sfiber] =
CRACKSTABILITY(sigapp,sfiber,tempradius,KIc,Y,C,MAT,m,n,q,smatrix,Vf)
% avgrad=average radius of the fibers(can be set in variables program).
global avgrad
if ( q > n )
   q = n;
   fprintf('The matrix is fully cracked');
   MAT = [ones(m,q),C,ones(m,n-q)];
else
   oldcrklen=q;
   % Now the task is to find the stability of the crack tip
   %The effective area of a fiber is a square element 4(avg. r) by
4(avg. r)
   %Calculate the area of the bridging zone
   area = q*m*((4*avgrad)^2);
   %Calculate the fiber cross sectional area for each fiber in the
bridging zone
   areafib = pi.*( tempradius(:,1:q).^2 );
   %Calculate the TOTAL stress in the bridging zone (TOTAL of all fiber
   %forces divided by the area of the bridging zone
   sigfib = (sum(sum( sfiber(:,1:q).*areafib )))/area;
   %Superimpose stresses to determine the effect of bridging zone
   sigeff = sigapp - sigfib;
   % To find the crack length
   cracklength = (q)*(4*avgrad);
   % Finding the effective stress intensity and comparing it to
   % critical stress intensity factor
   KIeff  = Y*sigeff*sqrt(pi*cracklength);
   if (KIeff < KIc)
      % fprintf('The crack is stable');
      stable = 1;
   else
      stable = 0;
      while (KIeff >= KIc)
         q=q+1;
         if ( q > n )
            q = n;
            % fprintf('The matrix is fully cracked');
            break;
         end
         area = q*m*((4*avgrad)^2);
         sigfib =
(sum(sum(sfiber(:,1:q).*(pi.*tempradius(:,1:q)))))/area;
         sigeff = sigapp - sigfib;
         cracklength = (q)*(4*avgrad);
         KIeff  = Y*sigeff*sqrt(pi*cracklength);
         if (KIeff >= KIc)
            % fprintf('The crack is made stable');
            MAT = [ones(m,q),C,ones(m,n-q)];
            break;
         end
      end
```

```
   end
end
%====================================================
% Here we need to redistribute the force taken by the matrix, which has
freshly cracked.
sfiber(:,oldcrklen+1:q)=sfiber(:,oldcrklen+1:q)+smatrix*(1-Vf)/Vf;
```

```
%**************************************************
%FRACBROC_DIST.m
function [sfiber,Exp,tempradius,whenbroketemp] =
FRACBROC_DIST(sfiber,strength,radius,tempradius,Exp,fdist,m,n,timestep)
% This part of the program calculates the index of the fibers which are
broken
% and then redistributes the load
%==========================================================
for b = 1:n
   for a = 1:m
      whenbroketemp(a,b)=-1;
      if (sfiber(a,b)~=0)
         if (sfiber(a,b) >= strength(a,b))
            whenbroketemp(a,b)=timestep;
            %fprintf('The index of the fiber failed is (%d,%d)\n',a,b);
            %fprintf('\n');
            % This function call will redistribute the force previously
taken by broken fiber.
            [sfiber] =
fordist(sfiber,tempradius,fdist,m,n,a,b,timestep,strength);
            Exp(a,b) = 0; % for records of the broken fiber
            sfiber(a,b) = 0; % ditto
            tempradius(a,b) = 0; %ditto
         end
      end

   end
   % fprintf('\n');
end
```

```
%***************************************************
%fiberdistance.m
function d = fiberdistance(i1,j1,i2,j2)
%d is the distance between the two fibers @ (i1,j1) and (i2,j2)
d = sqrt( ((j2-j1)^2)+((i2-i1)^2) );
```

```
%**************************************************
%fordist.m
function [sfiber] =
fordist(sfiber,tempradius,fdist,m,n,a,b,timestep,strength)
% This code redistributes the force of the broken fiber to all
% the remaining fibers according to the force-distribution law
%-----------------------------------------------------
% VARIABLES USED in this function:
% Fbroken=force that has to be redistributed from the broken fibre.
% dist= fiber distance from the broken fibre.
% fdist=force distribution exponents(randomly generated)
% distfactor= force distribution factor( proprtional to the load taken
by each fibre)
% denom= It is the sum of distribution factors.
%-----------------------------------------------------

Fbroken = sfiber(a,b).* (pi*(tempradius(a,b)^2));
%Calculate the distance^n matrix
for i=1:m
   for j=1:n
      if(sfiber(i,j)~= 0.0)
         dist=fiberdistance(a,b,i,j);
         if i==a &j==b
            distfactor(i,j)=0;
         else
            distfactor(i,j)=1/dist.^fdist(a,b);
         end
      else
         distfactor(i,j)=0;
      end
   end
end
%Calculate the sum of the inverse distance matrix
denom=sum(sum(distfactor));
Fij=zeros(m,n);
% Fij is the force being distributed to each fiber
for i=1:m
   if denom ==0.0
      fprintf('\nAll the fibres are broken at %d\n',timestep)
      break;
   end

   for j=1:n
      Fij(i,j)=Fbroken.*(distfactor(i,j)/denom);
   end
  end
% Final stresses on the individual fibers after the force from
% broken fiber is being redistributed.
for i=1:m
   for j=1:n
      if(sfiber(i,j)~=0)
         if (tempradius(i,j)>1e-20)

            sfiber(i,j)=sfiber(i,j)+
(Fij(i,j)/(pi*tempradius(i,j)))*(1/tempradius(i,j));
         end
      end
```

```
   end
end
sfiber(a,b) = 0;
```

```
%***************************************
%percentbroc.m
function p = percentbroc(Exp,m,n)
% This function calculates the percentage of fibers broken at a
particular
% timestep
num = (m*n)-(sum(sum(Exp)));
den = m*n;
p = (num/den)*100;
```

```
%************************************************************
%creep.m
function [sfiber,radius,tempradius]
=creep(sfiber,radius,tempradius,m,n,q,avgmod,timestep)
% This function m-file will apply creep to all the fibres
% convfctr= compensation factor from real time to timesteps
%========================================================
global sigapp Vf appldstress convfctr strsexponent
%---------------------------------------------------------
% we will calculate the  PRIMARY CREEP RATE here.
% It is given by de/dt(primary)=const1*stress^n*time ^m.
% Parameters taken from J.W.Holmes paper in j of matl sci,1992 vol27.
% valid in a range of 70-110 MPa.
const1=7.2e-15;
expnt1=1;
expnt2=-0.667;
pstrainrate=(const1*appldstress^expnt1*(timestep*convfctr)^expnt2)*conv
fctr;
%---------------------------------------------------------
% Calculating the  STEADY CREEP STRAIN RATE by using the power law.
% steadystrainrate=const2*stress^n
% Parameters taken from J.W.Holmes paper in j of matl sci,1992 vol27.
% valid in a range of 70-110 MPa.
const2=2.833e-25;
%strsexponent=2;% this value is set from the varibles program
sstrainrate=const2*(appldstress)^strsexponent*convfctr;
% Total creep strain rate.
strainrate=pstrainrate+sstrainrate;
%---------------------------------------------------------
% creeps all the fibres.
newsfiber=zeros(m,n);
for c1=1:n
   for c2=1:m
      % ****here matrix creep may be accounted.*********
      if tempradius(c2,c1)~=0.0
         areafin=(pi*tempradius(c2,c1).^2)/(strainrate+1);

newsfiber(c2,c1)=sfiber(c2,c1)*(pi*tempradius(c2,c1)^2)/areafin;
         % Setting the new radius.
         tempradius(c2,c1)=sqrt(areafin/(pi));
      end
   end
end
sfiber = newsfiber;
```

```
%***********************************************************
%convAnalysis.m
% This program is intended to generate the required form of the results
% by running the adopted "main" simulation from the "baremain.m".
% ********TESTING THE CONVERGENCE OF LIFETIMES******
% **** PLOTTING LIFETIME (VS) NO OF SIMULATIONS.
%=====================================================================
stepno=[1 2 3 5 7 10 15 20 25 35 50 75 100 150];% intended no of
simulations in each RUN(a total of 498 simulations).
cumtimestep=zeros(1,length(stepno));% Stores sum of all failure
timestep for each run
avtimestep=zeros(1,length(stepno));% Stores average failure timestep
for each run
stdtimestep=zeros(1,length(stepno));% Stores standard deviation of
timesteps for each run
ftimestep=zeros(length(stepno),max(stepno));% Stores no of timesteps of
each simulation
for otrcnt=1:length(stepno)
   for inrcnt=1:stepno(otrcnt)
      fprintf('RUN %d simulation %d',otrcnt,inrcnt)
      timestep=bareMain;% "main" program without animation and other
non essential things.
      ftimestep(otrcnt,inrcnt)=timestep; % failure timestep recording
for each simulation.
      cumtimestep(1,otrcnt)=cumtimestep(1,otrcnt)+timestep;
   end
   % storing the data.
   stdtimestep(1,otrcnt)=std(ftimestep(otrcnt,1:stepno(otrcnt)));
   avtimestep(1,otrcnt)=cumtimestep(1,otrcnt)/stepno(1,otrcnt);
   fileid=strcat('conAnalysis\RUN',num2str(otrcnt),'.txt');
   dlmwrite(fileid,[stdtimestep(1,otrcnt)';avtimestep(1,otrcnt)']);
   fileid2=strcat('conAnalysis\RUN',num2str(otrcnt),'failure.txt');
   dlmwrite(fileid2,ftimestep(otrcnt,1:stepno(otrcnt)));
end
dlmwrite('conAnalysis\FAILURETIMES.txt',ftimestep);
dlmwrite('conAnalysis\RESULTS.txt',[stdtimestep;avtimestep]);
figure(100)
errorbar(stepno,avtimestep,stdtimestep);
xlabel('Number of simulations ran');
ylabel('Average life time (in Time Steps)');
title('LIFETIME (VS) NO OF SIMULATIONS')
figfilename=strcat('conAnalysis.jpg');
saveas(100,figfilename);
stdtimestep
avtimestep
```

```
%**************************************************
%sim_Vf.m
% This program is intended to generate the required form of the results
% by running the adopted "main" simulation from the "baremain.m".
% ********TESTING THE EFFECT OF FIBER VOLUME FRACTION ON THE LIFE
TIME******
% **** PLOTTING LIFETIME (VS) FIBER VOLUME FRACTION.
%=====================================================================
global Vf
% values of fiber average modulus in GPa at which simulations are run
Volume_fraction=[.1 .2 .3 .4 .5 .6 .7 .8 .9];
ftimestep=zeros(50,length(Volume_fraction));% matrix to record the
timestep at failure.
for i=1:length(Volume_fraction)
   for j= 1:50
      fprintf('Modulus %f RUN %d',Volume_fraction(i),j);
     Vf= Volume_fraction(1,i);
      [timestep,tstep,cracklen1,perbroc]=baremain; % "main" program
without animation and other non essential things.
      ftimestep(j,i)=timestep;% record

fileid=strcat('results/Vf/Results_at',num2str(Volume_fraction(i)),'run_
',num2str(j),'.txt');
      dlmwrite(fileid,[tstep'  cracklen1' perbroc']);% write the
results in files for safety.
   end

fileid0=strcat('results/Vf/failure_at',num2str(Volume_fraction(i)),'.tx
t');
   dlmwrite(fileid0,ftimestep(:,i));% write the results in files for
safety.
end
fileid1=strcat('results/Vf/failureData(Vf).txt');
dlmwrite(fileid1,ftimestep);%write the results in files for safety.
%=====================================================================
```